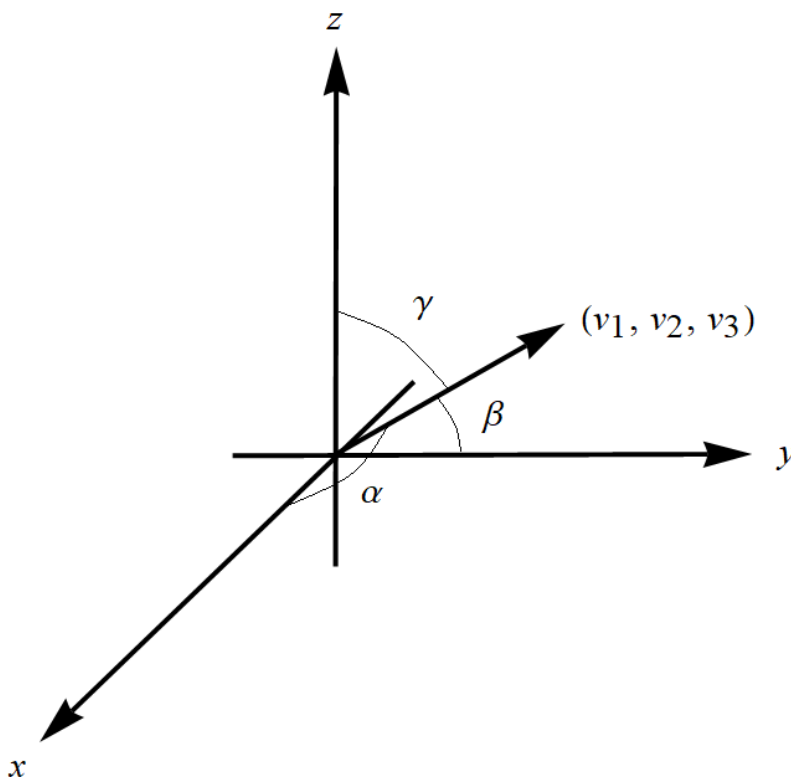


Exercise 28

For $\mathbf{v} = (v_1, v_2, v_3)$ let α, β, γ denote the angles between \mathbf{v} and the $x, y,$ and z axes, respectively. Show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.

Solution



From the figure,

$$\begin{aligned}\cos \alpha &= \frac{v_1}{\|\mathbf{v}\|} \\ \cos \beta &= \frac{v_2}{\|\mathbf{v}\|} \\ \cos \gamma &= \frac{v_3}{\|\mathbf{v}\|}.\end{aligned}$$

Therefore,

$$\begin{aligned}\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma &= \left(\frac{v_1}{\|\mathbf{v}\|}\right)^2 + \left(\frac{v_2}{\|\mathbf{v}\|}\right)^2 + \left(\frac{v_3}{\|\mathbf{v}\|}\right)^2 \\ &= \frac{v_1^2}{\|\mathbf{v}\|^2} + \frac{v_2^2}{\|\mathbf{v}\|^2} + \frac{v_3^2}{\|\mathbf{v}\|^2} \\ &= \frac{v_1^2 + v_2^2 + v_3^2}{\|\mathbf{v}\|^2} \\ &= \frac{\|\mathbf{v}\|^2}{\|\mathbf{v}\|^2} = 1.\end{aligned}$$