

Exercise 1

Verify that interchanging the first two rows of the 3×3 determinant

$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 2 & 0 & 2 \end{vmatrix}$$

changes the sign of the determinant.

Solution

Calculate the determinant as it's shown.

$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 2 & 0 & 2 \end{vmatrix} = -2 \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} = -2[(3)(2) - (1)(2)] = -2(6 - 2) = -2(4) = -8$$

Now switch the first two rows and calculate the determinant again.

$$\begin{vmatrix} 3 & 0 & 1 \\ 1 & 2 & 1 \\ 2 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} = 2[(3)(2) - (1)(2)] = 2(6 - 2) = 2(4) = 8$$

This verifies the fact that interchanging rows changes the sign of the determinant.