Exercise 11

In Exercises 9 to 12, describe all unit vectors orthogonal to both of the given vectors.

$$-5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}, \; 7\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}$$

Solution

Each of the vectors can be written as

$$\begin{aligned} -5\hat{\mathbf{x}} + 9\hat{\mathbf{y}} - 4\hat{\mathbf{z}} &= (-5, 9, -4) \\ 7\hat{\mathbf{x}} + 8\hat{\mathbf{y}} + 9\hat{\mathbf{z}} &= (7, 8, 9). \end{aligned}$$

Take the cross product of these two to obtain a vector orthogonal to both of them.

$$\begin{aligned} (-5\hat{\mathbf{x}} + 9\hat{\mathbf{y}} - 4\hat{\mathbf{z}}) \times (7\hat{\mathbf{x}} + 8\hat{\mathbf{y}} + 9\hat{\mathbf{z}}) &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ -5 & 9 & -4 \\ 7 & 8 & 9 \end{vmatrix} \\ &= \begin{vmatrix} 9 & -4 \\ 8 & 9 \end{vmatrix} \hat{\mathbf{x}} - \begin{vmatrix} -5 & -4 \\ 7 & 9 \end{vmatrix} \hat{\mathbf{y}} + \begin{vmatrix} -5 & 9 \\ 7 & 8 \end{vmatrix} \hat{\mathbf{z}} \\ &= (81 + 32)\hat{\mathbf{x}} - (-45 + 28)\hat{\mathbf{y}} + (-40 - 63)\hat{\mathbf{z}} \\ &= 113\hat{\mathbf{x}} + 17\hat{\mathbf{y}} - 103\hat{\mathbf{z}} \\ &= (113, 17, -103) \end{aligned}$$

To turn this vector into a unit vector, divide it by its magnitude.

$$\frac{(113, 17, -103)}{\sqrt{113^2 + 17^2 + (-103)^2}} = \frac{1}{\sqrt{23\,667}}(113, 17, -103)$$

There are two unit vectors orthogonal to $-5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$ and $7\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}$:

$$\pm\frac{1}{\sqrt{23\,667}}(113, 17, -103).$$