

Exercise 22

Find the intersection of the two planes with equations $3(x - 1) + 2y + (z + 1) = 0$ and $(x - 1) + 4y - (z + 1) = 0$.

Solution

The intersection for two planes is a straight line, which can be parameterized as

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where \mathbf{m} is the direction vector and \mathbf{b} is the position vector for any point on the line. The normal vectors to the given planes are obtained from the coefficients of x , y , and z : $(3, 2, 1)$ and $(1, 4, -1)$. Take the cross product of these two to find the direction vector of the line.

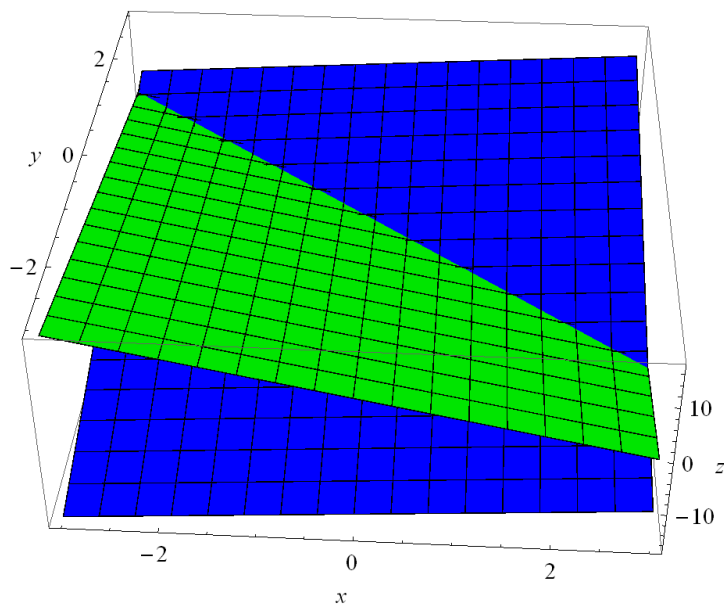
$$\mathbf{m} = (3, 2, 1) \times (1, 4, -1) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & 2 & 1 \\ 1 & 4 & -1 \end{vmatrix} = (-2-4)\hat{x} - (-3-1)\hat{y} + (12-2)\hat{z} = -6\hat{x} + 4\hat{y} + 10\hat{z} = (-6, 4, 10)$$

All that's left is to find a point common to both planes.

$$\left. \begin{aligned} 3(x - 1) + 2y + (z + 1) &= 0 \\ (x - 1) + 4y - (z + 1) &= 0 \end{aligned} \right\}$$

Choose $x = 1$, $y = 0$, and $z = -1$, for example. Then $\mathbf{b} = (1, 0, -1)$, and the line is

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{m}t + \mathbf{b} \\ &= (-6, 4, 10)t + (1, 0, -1) \\ &= (-6t + 1, 4t, 10t - 1). \end{aligned}$$



In green is $3(x - 1) + 2y + (z + 1) = 0$, and in blue is $(x - 1) + 4y - (z + 1) = 0$.