Exercise 28

Find an equation for the plane that passes through the point $(2, -1, 3)$ and is perpendicular to the line $\mathbf{v} = (1, -2, 2) + t(3, -2, 4)$.

Solution

The equation for a plane is

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0,$$

where $\mathbf{n}$ is a vector normal to the plane and $\mathbf{r}_0$ is the position vector for any point in the plane. The direction vector of the line, $(3, -2, 4)$, serves as a normal vector to the plane, and $(2, -1, 3)$ is the needed position vector.

$$(3, -2, 4) \cdot (x - 2, y + 1, z - 3) = 0$$

$$3(x - 2) - 2(y + 1) + 4(z - 3) = 0$$

$$3x - 6 - 2y - 2 + 4z - 12 = 0$$

$$3x - 2y + 4z = 20$$