Exercise 31

Find an equation for the plane containing the two (parallel) lines

$$\mathbf{v}_1 = (0, 1, -2) + t(2, 3, -1)$$

and

$$\mathbf{v}_2 = (2, -1, 0) + t(2, 3, -1).$$

Solution

The equation for a plane is

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0,$$

where $\mathbf{n}$ is a vector normal to the plane and $\mathbf{r}_0$ is the position vector for any point in the plane. To get the normal vector, take the cross product of the direction vector $(2, 3, -1)$ with the displacement vector,

$$(2, -1, 0) - (0, 1, -2) = (2, -2, 2).$$

Doing so gives

$$\mathbf{n} = (2, 3, -1) \times (2, -2, 2) = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 2 & 3 & -1 \\ 2 & -2 & 2 \end{vmatrix} = (6-2)\hat{\mathbf{x}} - (4+2)\hat{\mathbf{y}} + (-4-6)\hat{\mathbf{z}} = 4\hat{\mathbf{x}} - 6\hat{\mathbf{y}} - 10\hat{\mathbf{z}} = (4, -6, -10).$$

Either of the position vectors, $(2, -1, 0)$ or $(0, 1, -2)$, will do for $\mathbf{r}_0$. Choose $\mathbf{r}_0 = (2, -1, 0)$.

$$(4, -6, -10) \cdot (x - 2, y + 1, z - 0) = 0$$

$$4(x - 2) - 6(y + 1) - 10(z - 0) = 0$$

$$4x - 8 - 6y - 6 - 10z = 0$$

$$4x - 6y - 10z = 14$$

$$2x - 3y - 5z = 7$$