Exercise 39

Determine the distance from the plane $12x + 13y + 5z + 2 = 0$ to the point $(1, 1, -5)$.

Solution

The normal vector to the plane $\mathbf{n}$ is obtained from the coefficients of x, y, and z: $\mathbf{n} = (12, 13, 5)$. An equation for the line with direction vector $(12, 13, 5)$ that passes through $(1, 1, -5)$ is

$$\begin{aligned}\mathbf{y}(t) &= (12, 13, 5)t + (1, 1, -5) \\ &= (12t, 13t, 5t) + (1, 1, -5) \\ &= (12t + 1, 13t + 1, 5t - 5).\end{aligned}$$

Substitute $x = 12t + 1$, $y = 13t + 1$, and $z = 5t - 5$ into the equation for the plane and solve for t to find when the line intersects the plane.

$$12(12t + 1) + 13(13t + 1) + 5(5t - 5) + 2 = 0 \quad \rightarrow \quad t = -\frac{1}{169}$$

The point at which the line intersects the plane is then

$$\mathbf{y}\left(-\frac{1}{169}\right) = \left(12\frac{-1}{169} + 1, 13\frac{-1}{169} + 1, 5\frac{-1}{169} - 5\right) = \left(\frac{157}{169}, \frac{12}{13}, -\frac{850}{169}\right).$$

Therefore, the perpendicular distance from $(1, 1, -5)$ to the plane is

$$d = \sqrt{\left(1 - \frac{157}{169}\right)^2 + \left(1 - \frac{12}{13}\right)^2 + \left(-5 + \frac{850}{169}\right)^2} = \frac{\sqrt{2}}{13}.$$