

## Exercise 41

- (a) In mechanics, the **moment**  $M$  of a force  $\mathbf{F}$  about a point  $O$  is defined to be the magnitude of  $\mathbf{F}$  times the perpendicular distance  $d$  from  $O$  to the line of action of  $\mathbf{F}$ . The **vector moment**  $\mathbf{M}$  is the vector of magnitude  $M$  whose direction is perpendicular to the plane of  $O$  and  $\mathbf{F}$ , determined by the right-hand rule. Show that  $\mathbf{M} = \mathbf{R} \times \mathbf{F}$ , where  $\mathbf{R}$  is any vector from  $O$  to the line of action of  $\mathbf{F}$ . (See Figure 1.3.10.)

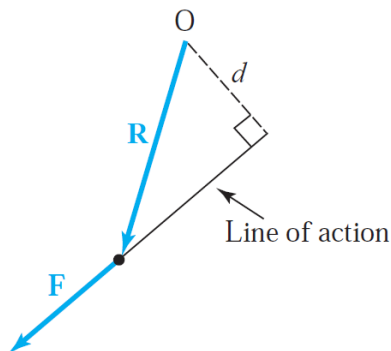


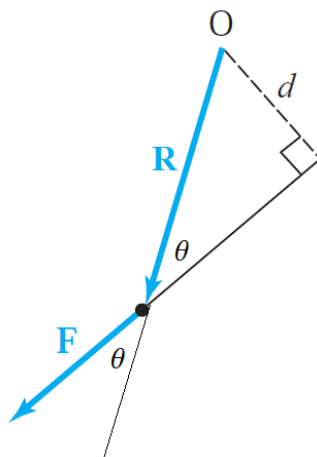
figure 1.3.10 Moment of a force.

- (b) Find the moment of the force vector  $\mathbf{F} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$  newtons about the origin if the line of action is  $x = 1 + t$ ,  $y = 1 - t$ ,  $z = 2t$ .

### Solution

#### Part (a)

Let  $\theta$  be the angle between  $\mathbf{R}$  and  $\mathbf{F}$ .



From the figure,

$$\sin \theta = \frac{d}{\|\mathbf{R}\|},$$

which means the perpendicular distance is

$$d = \|\mathbf{R}\| \sin \theta.$$

According to the definition, the moment  $M$  is the magnitude of  $\mathbf{F}$  times  $d$ .

$$M = \|\mathbf{F}\|d = \|\mathbf{F}\|\|\mathbf{R}\| \sin \theta$$

The right side is how the magnitude of the cross product is defined.

$$M = \|\mathbf{F} \times \mathbf{R}\| = \|\mathbf{R} \times \mathbf{F}\|$$

Since the vector moment's direction is perpendicular to the plane containing  $\mathbf{R}$  and  $\mathbf{F}$ , the cross product of  $\mathbf{R}$  and  $\mathbf{F}$  will give the correct direction.

$$\mathbf{M} = \pm(\mathbf{R} \times \mathbf{F})$$

To follow the right-hand corkscrew rule, the positive sign is chosen.

$$\mathbf{M} = \mathbf{R} \times \mathbf{F}$$

### Part (a)

The moment  $M$  is the magnitude of  $\mathbf{F}$  times the perpendicular distance from the origin to the line of action. The magnitude of  $\mathbf{F} = (1, -1, 2)$  is

$$\|\mathbf{F}\| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6}.$$

The distance from the origin to the line of action ( $x = 1 + t$ ,  $y = 1 - t$ ,  $z = 2t$ ) is

$$d(t) = \sqrt{[(1+t) - 0]^2 + [(1-t) - 0]^2 + [(2t) - 0]^2} = \sqrt{6t^2 + 2}.$$

The minimum of  $d(t)$  is the perpendicular distance:  $d_{\perp} = \sqrt{2}$ . Therefore,

$$M = \|\mathbf{F}\|d_{\perp} = \sqrt{6}\sqrt{2} = \sqrt{12}.$$