

## Exercise 8

What is the volume of the parallelepiped with sides  $\mathbf{i}$ ,  $3\mathbf{j} - \mathbf{k}$ , and  $4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ ?

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### Solution

Label each of the sides as

$$\begin{aligned}\mathbf{a} &= (1, 0, 0) \\ \mathbf{b} &= (0, 3, -1) \\ \mathbf{c} &= (4, 2, -1)\end{aligned}$$

The volume of the parallelepiped formed by these vectors is given by the triple product,

$$\begin{aligned}\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) &= (1, 0, 0) \cdot \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 0 & 3 & -1 \\ 4 & 2 & -1 \end{vmatrix} \\ &= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 4 & 2 & -1 \end{vmatrix} \\ &= 1 \begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix} - 0 + 0 \\ &= 1[(3)(-1) - (-1)(2)] \\ &= -1,\end{aligned}$$

or rather its magnitude:  $V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| = 1$ .