

Exercise 9

In Exercises 9 to 12, describe all unit vectors orthogonal to both of the given vectors.

$$\mathbf{i}, \mathbf{j}$$

Solution

Each of the vectors can be written as

$$\begin{aligned}\hat{\mathbf{x}} &= (1, 0, 0) \\ \hat{\mathbf{y}} &= (0, 1, 0).\end{aligned}$$

Take the cross product of these two to obtain a vector orthogonal to both of them.

$$\begin{aligned}\hat{\mathbf{x}} \times \hat{\mathbf{y}} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \\ &= \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} \hat{\mathbf{x}} - \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \hat{\mathbf{y}} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \hat{\mathbf{z}} \\ &= (0 - 0)\hat{\mathbf{x}} - (0 - 0)\hat{\mathbf{y}} + (1 - 0)\hat{\mathbf{z}} \\ &= \hat{\mathbf{z}} \\ &= (0, 0, 1)\end{aligned}$$

There are two unit vectors orthogonal to \mathbf{i} and \mathbf{j} : $\pm(0, 0, 1)$.