

## Exercise 10

Describe the following solids using inequalities. State the coordinate system used.

- (a) A cylindrical shell 8 units long, with inside diameter 2 units and outside diameter 3 units
- (b) A spherical shell with inside radius 4 units and outside radius 6 units
- (c) A hemisphere of diameter 5 units
- (d) A cube of side length 2

---

### Solution

#### Part (a)

Use a cylindrical coordinate system  $(r, \theta, z)$  centered on the cylinders' common axis at the midway point.

$$1 \leq r \leq 1.5, \quad 0 \leq \theta \leq 2\pi, \quad -4 \leq z \leq 4$$

#### Part (b)

Use a spherical coordinate system  $(\rho, \theta, \phi)$  centered at the spheres' common center.

$$4 \leq \rho \leq 6, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq \phi \leq \pi$$

#### Part (c)

Use a spherical coordinate system  $(\rho, \theta, \phi)$  centered at the hemisphere's center.

$$0 \leq \rho \leq 2.5, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq \phi \leq \frac{\pi}{2}$$

#### Part (d)

Use a Cartesian coordinate system  $(x, y, z)$  centered at one of the cube's edges.

$$0 \leq x \leq 2, \quad 0 \leq y \leq 2, \quad 0 \leq z \leq 2$$