

Exercise 4

Describe the geometric meaning of the following mappings in cylindrical coordinates:

(a) $(r, \theta, z) \mapsto (r, \theta, -z)$

(b) $(r, \theta, z) \mapsto (r, \theta + \pi, -z)$

(c) $(r, \theta, z) \mapsto (-r, \theta - \pi/4, z)$

Solution

Part (a)

This mapping reflects the point across the xy -plane.

Part (b)

This mapping rotates the point 180° around the z -axis (as indicated by the right-hand corkscrew rule) and then reflects it across the xy -plane. In other words, it reflects the point across the origin.

Part (c)

This mapping rotates the point 135° around the z -axis (as indicated by the right-hand corkscrew rule). Note that changing r to $-r$ is effectively the same as adding π to θ . In total, the change in θ is $\pi - \pi/4 = 3\pi/4 = 135^\circ$.