

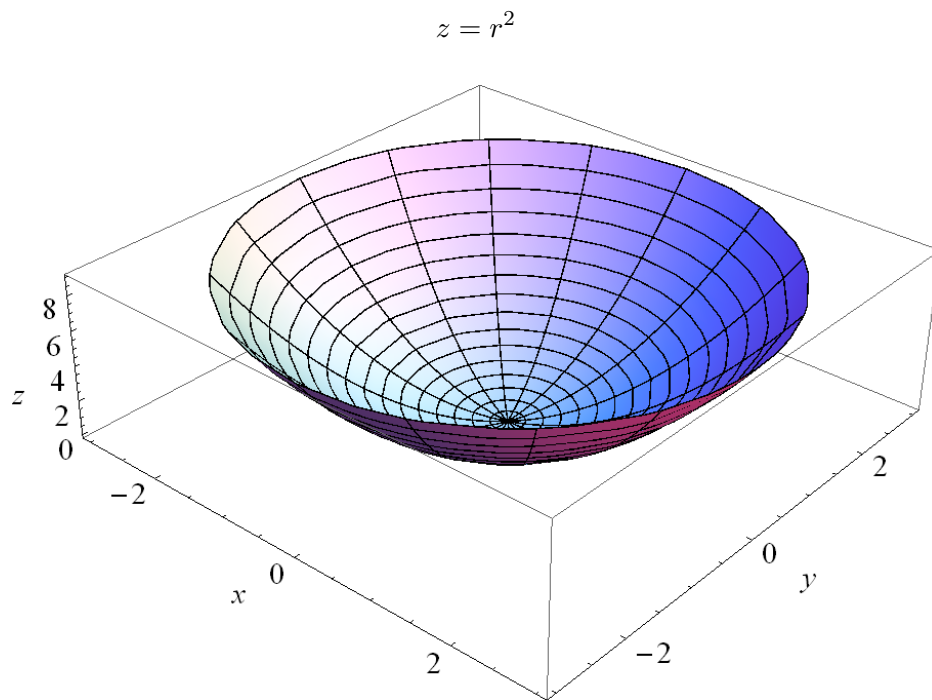
Exercise 7

Sketch the following surfaces:

- (a) $z = r^2$
- (b) $\rho = 4 \csc \phi \sec \theta$
- (c) $r = 4 \sin \theta$
- (d) $\rho \sin \phi = 2$

Solution

Part (a)



Part (b)

$$\rho = 4 \csc \phi \sec \theta$$

Write the equation in terms of sine and cosine.

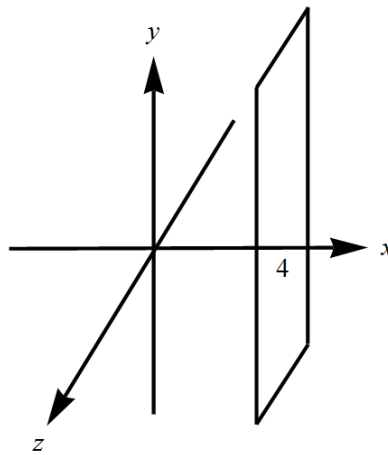
$$\rho = \frac{4}{\sin \phi \cos \theta}$$

Multiply both sides by $\sin \phi \cos \theta$.

$$\rho \sin \phi \cos \theta = 4$$

Substitute $x = \rho \sin \phi \cos \theta$.

$$x = 4$$



Part (c)

$$r = 4 \sin \theta$$

Change this to Cartesian coordinates by substituting $r = \sqrt{x^2 + y^2}$ and $y = r \sin \theta$.

$$\sqrt{x^2 + y^2} = 4 \left(\frac{y}{\sqrt{x^2 + y^2}} \right)$$

Multiply both sides by $\sqrt{x^2 + y^2}$.

$$x^2 + y^2 = 4y$$

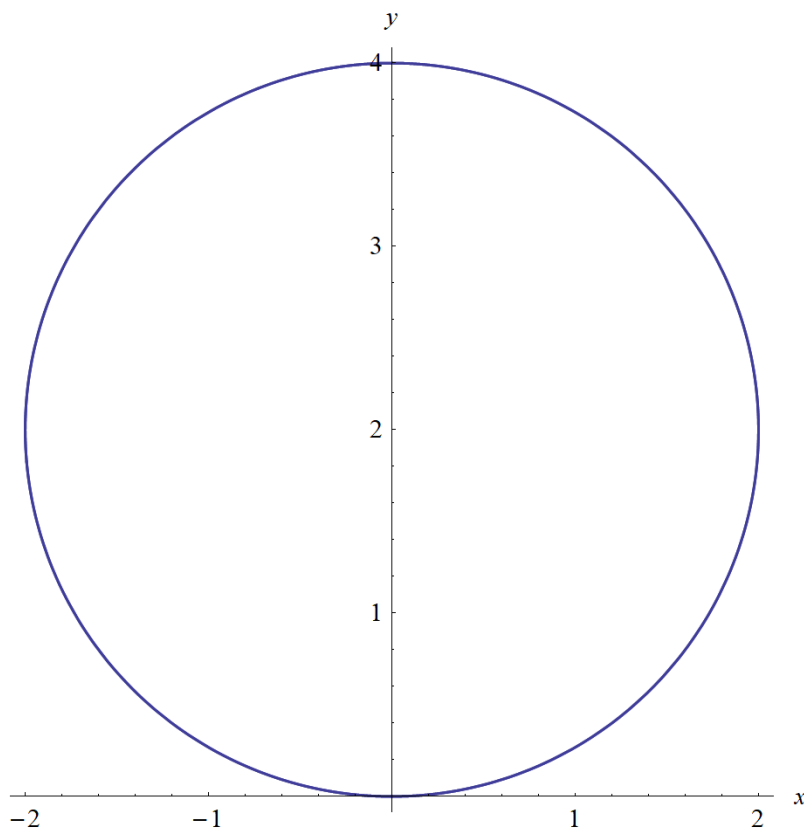
Bring $4y$ to the left side and complete the square.

$$x^2 + y^2 - 4y = 0$$

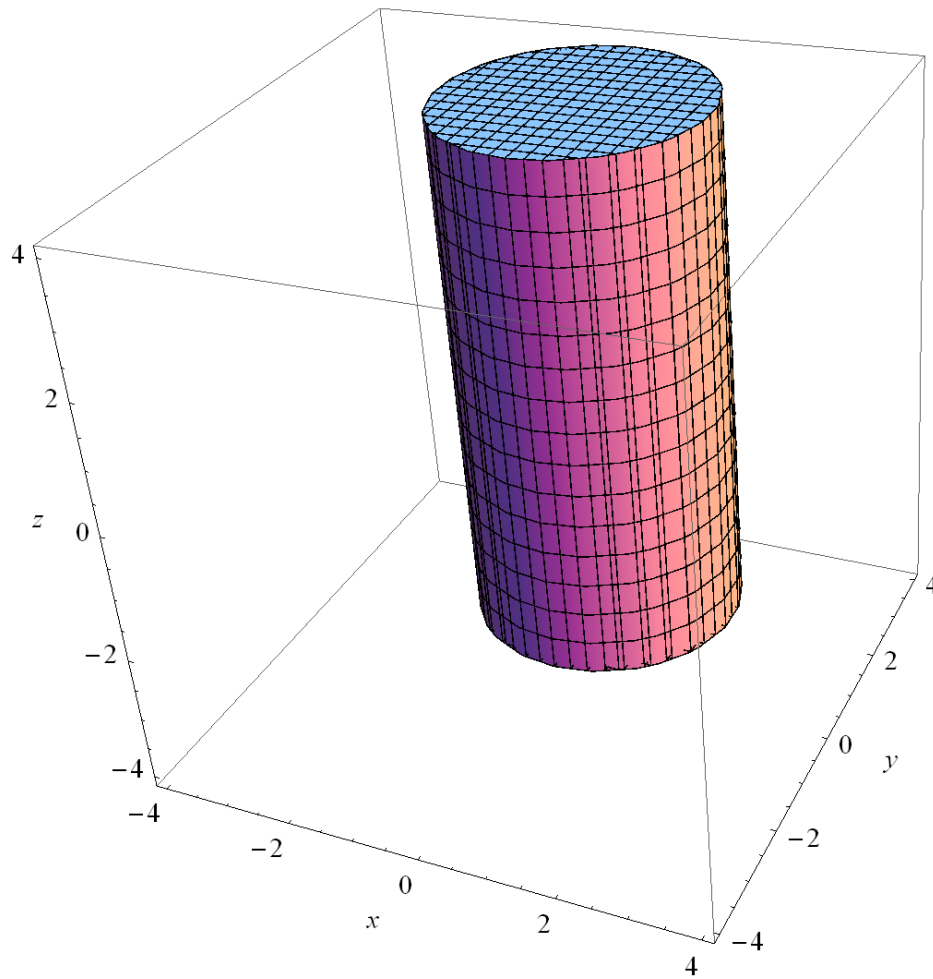
$$x^2 + y^2 - 4y + 4 = 4$$

$$x^2 + (y - 2)^2 = 4$$

This is a circle centered at $(0, 2)$ with a radius of 2.



In three dimensions this is actually a cylinder that extends indefinitely in the z -direction.



Part (d)

$$\rho \sin \phi = 2$$

Square both sides.

$$\rho^2 \sin^2 \phi = 4$$

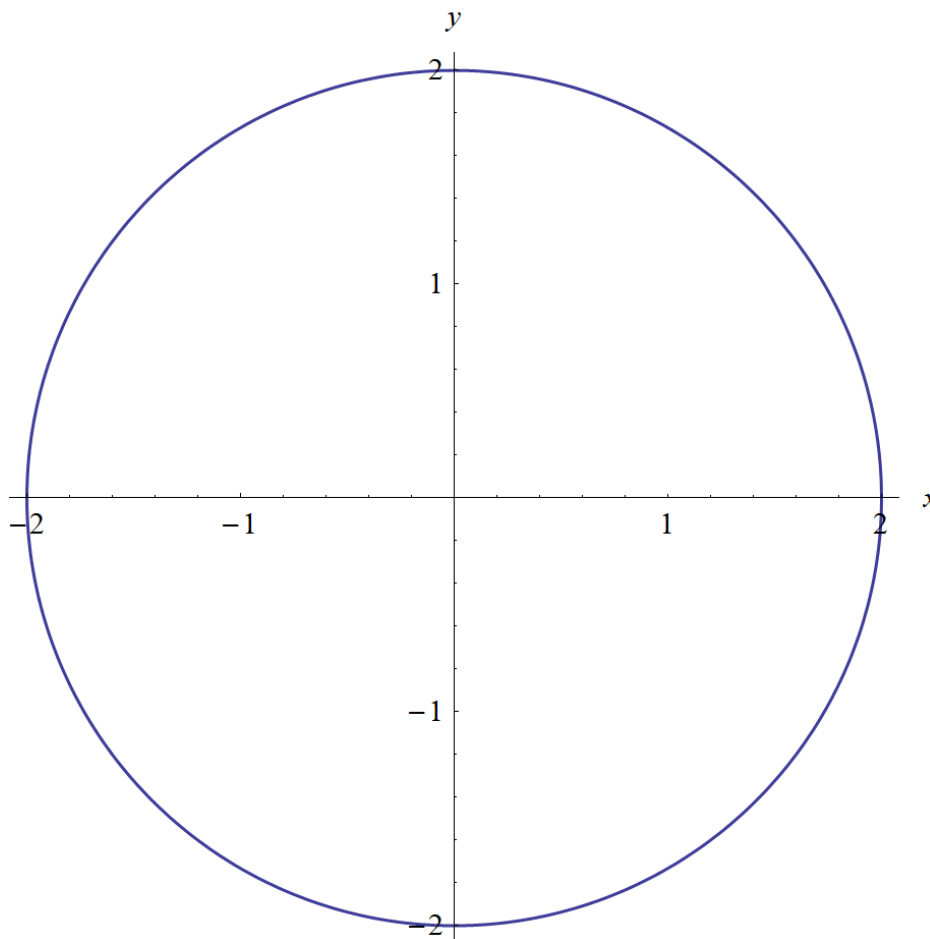
$$\rho^2 \sin^2 \phi (\cos^2 \theta + \sin^2 \theta) = 4$$

$$\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta = 4$$

Substitute $x = \rho \sin \phi \cos \theta$ and $y = \rho \sin \phi \sin \theta$.

$$x^2 + y^2 = 4$$

This is a circle centered at $(0, 0)$ with radius 2.



In three dimensions this is actually a cylinder that extends indefinitely in the z -direction.

