Problem 1-10

Express the Planck distribution law in terms of λ (and $d\lambda$) by using the relationship $\lambda\nu = c$.

Solution

The Planck distribution law for blackbody radiation is given by

$$\rho_\nu(T)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/k_BT}-1}\,d\nu.$$

Use the chain rule to write this in terms of the wavelength λ.

$$\rho_\nu(T)\left(\frac{d\nu}{d\lambda}\right)d\lambda = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/k_BT}-1}\left(\frac{d\nu}{d\lambda}\right)d\lambda \tag{1}$$

The relationship between frequency and wavelength for blackbody radiation is

$$\lambda\nu = c \quad \Rightarrow \quad \begin{cases} \nu = \dfrac{c}{\lambda} \\ \dfrac{d\nu}{d\lambda} = -\dfrac{c}{\lambda^2} \end{cases}.$$

As a result, equation (1) becomes

$$\begin{aligned}[-\rho_\lambda(T)]\,d\lambda &= \frac{8\pi h}{c^3}\frac{\left(\frac{c}{\lambda}\right)^3}{\exp\left(\frac{hc}{k_BT\lambda}\right)-1}\left(-\frac{c}{\lambda^2}\right)d\lambda \\ &= -\frac{8\pi hc}{\lambda^5}\frac{d\lambda}{\exp\left(\frac{hc}{k_BT\lambda}\right)-1}.\end{aligned}$$

Therefore, the Planck distribution in terms of wavelength is

$$\rho_\lambda(T)\,d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{\exp\left(\frac{hc}{k_BT\lambda}\right)-1}.$$