

Exercise 14

For the following exercises, find the domain, range, and all zeros/intercepts, if any, of the functions.

$$f(x) = \frac{x}{x^2 - 16}$$

Solution

$f(x)$ is a rational function, so the domain is the set of all x except the values where the denominator is zero.

$$x^2 - 16 \neq 0$$

$$(x + 4)(x - 4) \neq 0$$

$$x \neq \{-4, 4\}$$

Therefore, the domain is $\{x \mid x \neq \pm 4\}$. $x = -4$ and $x = 4$ are the vertical asymptotes. $f(x)$ is continuous between these vertical asymptotes, so $f(x)$ takes on all values between

$$f(-3.999) = \frac{-3.999}{(-3.999)^2 - 16} \approx 499.9$$

$$f(0) = 0$$

$$f(3.999) = \frac{3.999}{(3.999)^2 - 16} \approx -499.9.$$

Choosing values even closer to ± 4 , such as $x = -3.99999$ or $x = +3.99999$, gives even larger values for $f(x)$. Therefore, the range is $\{y \mid -\infty < y < \infty\}$. Zeros occur where the numerator is zero.

$$f(x) = \frac{x}{x^2 - 16} = 0 \quad \Rightarrow \quad x = 0$$

The one x - and y -intercept is $(0, 0)$.

Below is a graph of $f(x)$ versus x to confirm these results.

