

## Exercise 19

For the following exercises, find the domain, range, and all zeros/intercepts, if any, of the functions.

$$g(x) = \frac{3}{x - 4}$$

### Solution

The domain is the set of all  $x$  where the denominator is not zero.

$$x - 4 \neq 0$$

$$x \neq 4$$

Therefore, the domain is  $\{x \mid x \neq 4\}$ . There's a vertical asymptote at  $x = 4$ .  $f(x)$  is continuous for  $x \neq 4$ , so it takes on all values between

$$g(4.0001) = \frac{3}{4.0001 - 4} \approx 30\,000$$

$$g(1000) = \frac{3}{1000 - 4} \approx 0.003$$

and all values between

$$g(3.9999) = \frac{3}{-3.9999 - 4} \approx -30\,000$$

$$g(-1000) = \frac{3}{-1000 - 4} \approx -0.003.$$

Putting in values of  $x$  even closer to 4 yields even higher values, and putting in values of  $x$  larger than 1000 yields a number even closer to zero. Therefore, the range is  $\{y \mid y \neq 0\}$ . Plug in  $x = 0$  to the function:  $g(0) = -3/4$ . The  $y$ -intercept is then  $(0, -3/4)$ . Below is a graph of  $g(x)$  versus  $x$  to confirm these results.

