

## Exercise 1

State whether the given sums are equal or unequal.

a.  $\sum_{i=1}^{10} i$  and  $\sum_{k=1}^{10} k$

b.  $\sum_{i=1}^{10} i$  and  $\sum_{i=6}^{15} (i - 5)$

c.  $\sum_{i=1}^{10} i(i - 1)$  and  $\sum_{j=0}^9 (j + 1)j$

d.  $\sum_{i=1}^{10} i(i - 1)$  and  $\sum_{k=1}^{10} (k^2 - k)$

### Solution

#### Part (a)

$\sum_{i=1}^{10} i$  and  $\sum_{k=1}^{10} k$  are equal because  $i$  and  $k$  are dummy indices. The substitution  $k = i$  can be made to turn one sum into the other.

#### Part (b)

Make the substitution  $j = i$  in the first sum,

$$\sum_{i=1}^{10} i = \sum_{j=1}^{10} j,$$

and make the substitution  $j = i - 5$  in the second sum,

$$\sum_{i=6}^{15} (i - 5) = \sum_{j+5=6}^{15} j = \sum_{j=1}^{10} j.$$

Therefore, the two sums are equal.

#### Part (c)

Make the substitution  $k = i$  in the first sum,

$$\sum_{i=1}^{10} i(i - 1) = \sum_{k=1}^{10} k(k - 1),$$

and make the substitution  $k = j + 1$  in the second sum,

$$\sum_{j=0}^9 (j + 1)j = \sum_{k-1=0}^9 (k)(k - 1) = \sum_{k=1}^{10} k(k - 1).$$

Therefore, the two sums are equal.

**Part (d)**

The two sums are equal because  $i$  and  $k$  are dummy indices.

$$\sum_{i=1}^{10} i(i-1) = \sum_{i=1}^{10} (i^2 - i) = \sum_{k=1}^{10} (k^2 - k)$$