

Exercise 10

In the following exercises, use summation properties and formulas to rewrite and evaluate the sums.

$$\sum_{j=11}^{20} (j^2 - 10j)$$

Solution

$$\begin{aligned}\sum_{j=11}^{20} (j^2 - 10j) &= \sum_{j=11}^{20} j^2 - \sum_{j=11}^{20} 10j \\ &= \sum_{j=11}^{20} j^2 - 10 \sum_{j=11}^{20} j \\ &= \left(\sum_{j=1}^{20} j^2 - \sum_{j=1}^{10} j^2 \right) - 10 \left(\sum_{j=1}^{20} j - \sum_{j=1}^{10} j \right) \\ &= \left[\frac{20(20+1)(40+1)}{6} - \frac{10(10+1)(20+1)}{6} \right] - 10 \left[\frac{20(20+1)}{2} - \frac{10(10+1)}{2} \right] \\ &= (2870 - 385) - 10(210 - 55) \\ &= 935\end{aligned}$$