

Exercise 13

Let L_n denote the left-endpoint sum using n subintervals and let R_n denote the corresponding right-endpoint sum. In the following exercises, compute the indicated left and right sums for the given functions on the indicated interval.

$$R_4 \text{ for } g(x) = \cos(\pi x) \text{ on } [0, 1]$$

Solution

Since we're using the right-endpoint sum with $n = 4$ to approximate the integral of $g(x)$ from 0 to 1, the sum is taken from 1 to 4 rather than 0 to 3.

$$\begin{aligned} \int_0^1 g(x) dx &\approx \sum_{i=1}^4 g(x_i) \Delta x = \sum_{i=1}^4 \cos(\pi x_i) \Delta x \\ &= \sum_{i=1}^4 \cos[\pi(0 + i\Delta x)] \Delta x \\ &= \sum_{i=1}^4 \cos(\pi i \Delta x) \Delta x \\ &= \sum_{i=1}^4 \cos \left[\pi i \left(\frac{1-0}{4} \right) \right] \left(\frac{1-0}{4} \right) \\ &= \sum_{i=1}^4 \cos \left[\pi i \left(\frac{1}{4} \right) \right] \left(\frac{1}{4} \right) \\ &= \frac{1}{4} \sum_{i=1}^4 \cos \left(\frac{\pi i}{4} \right) \\ &= \frac{1}{4} \left[\cos \frac{\pi(1)}{4} + \cos \frac{\pi(2)}{4} + \cos \frac{\pi(3)}{4} + \cos \frac{\pi(4)}{4} \right] \\ &= \frac{1}{4} \left(\cos \frac{\pi}{4} + \cos \frac{\pi}{2} + \cos \frac{3\pi}{4} + \cos \pi \right) \\ &= \frac{1}{4} \left(\frac{1}{\sqrt{2}} + 0 - \frac{1}{\sqrt{2}} - 1 \right) \\ &= -\frac{1}{4} \end{aligned}$$