Exercise 15

Let L_n denote the left-endpoint sum using n subintervals and let R_n denote the corresponding right-endpoint sum. In the following exercises, compute the indicated left and right sums for the given functions on the indicated interval.

$$R_6 \text{ for } f(x) = \frac{1}{x(x-1)} \text{ on } [2, 5]$$

Solution

Since we're using the right-endpoint sum with $n = 6$ to approximate the integral of $f(x)$ from 2 to 5, the sum is taken from 1 to 6 rather than 0 to 5.

$$\begin{aligned}
\int_2^5 f(x)\,dx \approx \sum_{i=1}^{6} f(x_i)\Delta x &= \sum_{i=1}^{6} \frac{1}{x_i(x_i-1)}\Delta x \\
&= \sum_{i=1}^{6} \frac{1}{(2+i\Delta x)[(2+i\Delta x)-1]}\Delta x \\
&= \sum_{i=1}^{6} \frac{1}{(2+i\Delta x)(1+i\Delta x)}\Delta x \\
&= \sum_{i=1}^{6} \frac{1}{\left[2+i\left(\frac{5-2}{6}\right)\right]\left[1+i\left(\frac{5-2}{6}\right)\right]}\left(\frac{5-2}{6}\right) \\
&= \sum_{i=1}^{6} \frac{1}{\left[2+i\left(\frac{1}{2}\right)\right]\left[1+i\left(\frac{1}{2}\right)\right]}\left(\frac{2}{4}\right) \\
&= 2\sum_{i=1}^{6} \frac{1}{(4+i)(2+i)} \\
&= 2\left[\frac{1}{(4+1)(2+1)} + \frac{1}{(4+2)(2+2)} + \frac{1}{(4+3)(2+3)}\right. \\
&\qquad \left. + \frac{1}{(4+4)(2+4)} + \frac{1}{(4+5)(2+5)} + \frac{1}{(4+6)(2+6)}\right] \\
&= 2\left(\frac{1}{15} + \frac{1}{24} + \frac{1}{35} + \frac{1}{48} + \frac{1}{63} + \frac{1}{80}\right) \\
&= 2\left(\frac{67}{360}\right) \\
&= \frac{67}{180}
\end{aligned}$$