Exercise 17

Let L_n denote the left-endpoint sum using n subintervals and let R_n denote the corresponding right-endpoint sum. In the following exercises, compute the indicated left and right sums for the given functions on the indicated interval.

$$L_4 \text{ for } \frac{1}{x^2+1} \text{ on } [-2, 2]$$

Solution

Since we're using the left-endpoint sum with $n = 4$ to approximate the integral of $1/(x^2+1)$ from -2 to 2, the sum is taken from 0 to 3 rather than 1 to 4.

$$\begin{aligned}
\int_{-2}^{2} \frac{1}{x^2+1}\,dx \approx \sum_{i=0}^{3} \frac{1}{x_i^2+1}\Delta x &= \sum_{i=0}^{3} \frac{1}{(-2+i\Delta x)^2+1}\Delta x \\
&= \sum_{i=0}^{3} \frac{1}{[4-4i\Delta x+i^2(\Delta x)^2]+1}\Delta x \\
&= \sum_{i=0}^{3} \frac{1}{5-4i\Delta x+i^2(\Delta x)^2}\Delta x \\
&= \sum_{i=0}^{3} \frac{1}{5-4i\left[\frac{2-(-2)}{4}\right]+i^2\left[\frac{2-(-2)}{4}\right]^2}\left[\frac{2-(-2)}{4}\right] \\
&= \sum_{i=0}^{3} \frac{1}{5-4i(1)+i^2(1)^2}(1) \\
&= \sum_{i=0}^{3} \frac{1}{5-4i+i^2} \\
&= \frac{1}{5-4(0)+(0)^2} + \frac{1}{5-4(1)+(1)^2} + \frac{1}{5-4(2)+(2)^2} + \frac{1}{5-4(3)+(3)^2} \\
&= \frac{1}{5} + \frac{1}{2} + 1 + \frac{1}{2} \\
&= \frac{11}{5}
\end{aligned}$$