Exercise 19

Let L_n denote the left-endpoint sum using n subintervals and let R_n denote the corresponding right-endpoint sum. In the following exercises, compute the indicated left and right sums for the given functions on the indicated interval.

$$L_8 \text{ for } x^2 - 2x + 1 \text{ on } [0, 2]$$

Solution

Since we're using the right-endpoint sum with $n = 8$ to approximate the integral of $x^2 - 2x + 1$ from 0 to 2, the sum is taken from 0 to 7 rather than 1 to 8.

$$\begin{aligned}
\int_0^2 (x^2 - 2x + 1)\,dx \approx \sum_{i=0}^{7}(x_i^2 - 2x_i + 1)\Delta x &= \sum_{i=0}^{7}[(0 + i\Delta x)^2 - 2(0 + i\Delta x) + 1]\Delta x \\
&= \sum_{i=0}^{7}[i^2(\Delta x)^2 - 2i\Delta x + 1]\Delta x \\
&= \sum_{i=0}^{7}\left[i^2\left(\frac{2-0}{8}\right)^2 - 2i\left(\frac{2-0}{8}\right) + 1\right]\left(\frac{2-0}{8}\right) \\
&= \sum_{i=0}^{7}\left[i^2\left(\frac{1}{4}\right)^2 - 2i\left(\frac{1}{4}\right) + 1\right]\left(\frac{1}{4}\right) \\
&= \sum_{i=0}^{7}\left[i^2\left(\frac{1}{4}\right)^3 - 2i\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)\right] \\
&= \sum_{i=0}^{7}\left[i^2\left(\frac{1}{64}\right) - 2i\left(\frac{1}{16}\right) + \left(\frac{1}{4}\right)\right] \\
&= \sum_{i=0}^{7} i^2\left(\frac{1}{64}\right) - \sum_{i=0}^{7} i\left(\frac{1}{8}\right) + \sum_{i=0}^{7}\left(\frac{1}{4}\right) \\
&= \frac{1}{64}\sum_{i=0}^{7} i^2 - \frac{1}{8}\sum_{i=0}^{7} i + \frac{1}{4}\sum_{i=0}^{7} 1 \\
&= \frac{1}{64}\left(0^2 + \sum_{i=1}^{7} i^2\right) - \frac{1}{8}\left(0 + \sum_{i=1}^{7} i\right) + \frac{1}{4}\left(1 + \sum_{i=1}^{7} 1\right) \\
&= \frac{1}{64}\left[\frac{7(7+1)(14+1)}{6}\right] - \frac{1}{8}\left[\frac{7(7+1)}{2}\right] + \frac{1}{4}(1 + 7 \cdot 1) \\
&= \frac{1}{64}(140) - \frac{1}{8}(28) + \frac{1}{4}(8) \\
&= \frac{11}{16}
\end{aligned}$$