Problem 1

Suppose that the universal set S is defined as $S = \{1, 2, \cdots, 10\}$ and $A = \{1, 2, 3\}$, $B = \{X \in S : 2 \leq X \leq 7\}$, and $C = \{7, 8, 9, 10\}$.

(a) Find $A \cup B$.

(b) Find $(A \cup C) - B$.

(c) Find $\overline{A} \cup (B - C)$.

(d) Do A, B, and C form a partition of S?

Solution

In English, B is the collection of elements in the universal set S that lie between 2 and 7 inclusive.

$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 4, 5, 6, 7\}$$

$$C = \{7, 8, 9, 10\}$$

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$A \cup B$ is the union of elements in sets A and B.

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

$(A \cup C) - B$ is the union of elements in sets A and C that do not also lie in B.

$$A \cup C = \{1, 2, 3, 7, 8, 9, 10\} \quad \rightarrow \quad (A \cup C) - B = \{1, 8, 9, 10\}$$

$\overline{A} \cup (B - C)$ is the union of elements in sets $\overline{A}$ and $B - C$. $\overline{A}$ is the set of elements in S that do not also lie in A. $B - C$ is the set of elements that lie in B that do not also lie in C.

$$\left.\begin{aligned} B - C &= \{2, 3, 4, 5, 6\} \\ \overline{A} &= \{4, 5, 6, 7, 8, 9, 10\} \end{aligned}\right\} \quad \rightarrow \quad \overline{A} \cup (B - C) = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

A, B, and C form a partition of S if they don't have elements in common and their union is S. Even though it's true that

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = S,$$

A and B and C are not disjoint, which means they do not form a partition of S.