

Multiples of 3 and 5

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

Solution

The multiples of 3 are $3n$, where $n = 1, 2, \dots$. Similarly, the multiples of 5 are $5n$. The highest multiple of 3 below 1000 is 999 (corresponding to $n = 333$), and the highest multiple of 5 below 1000 is 995 (corresponding to $n = 199$). The sum of all the multiples of 3 or 5 below 1000 is then

$$S = \sum_{n=1}^{333} 3n + \sum_{n=1}^{199} 5n - \sum_{n=1}^{66} (3 \times 5)n.$$

Numbers that have 3 and 5 as factors are counted twice; that's why multiples of 15 have to be subtracted from the total. Bring the constants in front.

$$S = 3 \sum_{n=1}^{333} n + 5 \sum_{n=1}^{199} n - 15 \sum_{n=1}^{66} n$$

Recall the known summation formula,

$$\sum_{n=1}^N n = \frac{N(N+1)}{2}.$$

Therefore,

$$S = 3 \cdot \frac{333(333+1)}{2} + 5 \cdot \frac{199(199+1)}{2} - 15 \cdot \frac{66(66+1)}{2} = 233\,168.$$