

Highly divisible triangular number

The sequence of triangle numbers is generated by adding the natural numbers. So the 7th triangle number would be $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. The first ten terms would be:

$$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \dots$$

Let us list the factors of the first seven triangle numbers:

1: 1
3: 1, 3
6: 1, 2, 3, 6
10: 1, 2, 5, 10
15: 1, 3, 5, 15
21: 1, 3, 7, 21
28: 1, 2, 4, 7, 14, 28

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

Solution

Note that the n th triangle number is given by the summation formula,

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}.$$

Running the accompanying C program for a while yields $n = 12\,375$ for the first triangle number with over 500 divisors. Plugging this into the formula gives

$$\frac{(12\,375)(12\,376)}{2} = 76\,576\,500$$

for the triangle number.