

## Sum square difference

The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + \dots + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + \dots + 10)^2 = 55^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is  $3025 - 385 = 2640$ .

Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

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### Solution

Use the known summation formulas,

$$\sum_{n=1}^N n = \frac{N(N+1)}{2}$$
$$\sum_{n=1}^N n^2 = \frac{N(N+1)(2N+1)}{6}.$$

The sum of the squares of the first hundred natural numbers is

$$S_1 = 1^2 + 2^2 + \dots + 100^2 = \sum_{n=1}^{100} n^2 = \frac{100(101)(201)}{6} = 338\,350.$$

The square of the sum of the first hundred natural numbers is

$$S_2 = (1 + 2 + \dots + 100)^2 = \left( \sum_{n=1}^{100} n \right)^2 = \left[ \frac{100(101)}{2} \right]^2 = 5050^2 = 25\,502\,500.$$

The difference of these sums is then

$$\Delta S = S_2 - S_1 = 25\,502\,500 - 338\,350 = 25\,164\,150.$$