Problem 1.1

Gravity vs. electricity

(a) In the domain of elementary particles, a natural unit of mass is the mass of a *nucleon*, that is, a proton or a neutron, the basic massive building blocks of ordinary matter. Given the nucleon mass as $1.67 \cdot 10^{-27}$ kg and the gravitational constant G as $6.67 \cdot 10^{-11}$ $\text{m}^3/(\text{kg s}^2)$, compare the gravitational attraction of two protons with their electrostatic repulsion. This shows why we call gravitation a very *weak* force.

(b) The distance between the two protons in the helium nucleus could be at one instant as much as 10^{-15} m. How large is the force of electrical repulsion between two protons at that distance? Express it in newtons, and in pounds. Even stronger is the *nuclear* force that acts between any pair of hadrons (including neutrons and protons) when they are that close together.

Solution

Part (a)

The gravitational force between two masses is given by Newton's law of gravitation, and the electric force between two charges is given by Coulomb's law.

$$F_g = G\frac{m_1 m_2}{r^2} \qquad\qquad F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

For two protons in particular,

$$F_g = G\frac{(m_p)(m_p)}{r^2} \qquad\qquad F_e = \frac{1}{4\pi\epsilon_0}\frac{(+e)(+e)}{r^2}.$$

Take the ratio of F_e and F_g to determine how much stronger the electric force is.

$$\begin{aligned}
\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\epsilon_0}\frac{(+e)(+e)}{r^2}}{G\frac{(m_p)(m_p)}{r^2}} &= \frac{1}{4\pi\epsilon_0 G}\frac{e^2}{m_p^2} \\
&\approx \frac{1}{4(3.14)(8.85\times 10^{-12})(6.67\times 10^{-11})}\frac{(1.60\times 10^{-19})^2}{(1.67\times 10^{-27})^2} \\
&\approx 1.24\times 10^{36}
\end{aligned}$$

Therefore, multiplying both sides by F_g,

$$F_e \approx (1.24\times 10^{36})F_g$$

the electric force between two protons is roughly 10^{36} times that of the gravitational force between them.

Part (b)

The electric force between two protons that are a distance $r = 10^{-15}$ m apart is

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{(+e)(+e)}{r^2} \approx \frac{1}{4(3.14)(8.85\times 10^{-12})}\frac{(1.60\times 10^{-19})^2}{(10^{-15})^2}\ \mathrm{N} \approx 231\ \mathrm{N}.$$

Multiply this result by the appropriate conversion factor to turn it into pounds.

$$F_e \approx 231\ \cancel{\mathrm{N}} \times \frac{0.224881\ \mathrm{lb}}{1\ \cancel{\mathrm{N}}} \approx 51.9\ \mathrm{lb}$$