Problem 10

In how many ways can 8 people be seated in a row if

(a) there are no restrictions on the seating arrangement?

(b) persons A and B must sit next to each other?

(c) there are 4 men and 4 women and no 2 men or 2 women can sit next to each other?

(d) there are 5 men and they must sit next to one another?

(e) there are 4 married couples and each couple must sit together?

Solution

Part (a)

Any of the eight people can be placed in the first spot. Only seven remain for the second spot and so on. By the counting principle, there are

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8! = 40\,320$$

different seating arrangements.

Part (b)

Treat persons A and B as a block. The number of ways to arrange the other six people and this block is $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$. Multiply this result by the number of ways persons A and B can be arranged inside the block to get the final answer.

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times (2 \times 1) = 10\,080$$

Part (c)

Any of the eight people can go in the first spot. Then any four members of the opposite sex can go in the second spot. Three members of the first sex are left for the third spot. Three members of the opposite sex are left for the fourth spot. Then only two members of the first sex are left for the fifth spot. Then only two members of the opposite sex are left for the sixth spot. And only one member of each sex is left for the seventh and eighth spots. By the counting principle, there are

$$8 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 1152$$

different ways to arrange the people this way.

Part (d)

Treat the five men as one block. The number of ways the three women and this block can be arranged is $4 \times 3 \times 2 \times 1$. Multiply this result by the number of ways the five men can be arranged inside the block to get the final answer.

$$4 \times 3 \times 2 \times 1 \times (5 \times 4 \times 3 \times 2 \times 1) = 2880$$

Part (e)

Treat the eight people as four blocks of two. The number of ways to arrange these four blocks is $4 \times 3 \times 2 \times 1$. Multiply this result by the number of ways to arrange the couples in each block to get the final answer.

$$4 \times 3 \times 2 \times 1 \times (2 \times 1) \times (2 \times 1) \times (2 \times 1) \times (2 \times 1) = 384$$