

Problem 4

John, Jim, Jay, and Jack have formed a band consisting of 4 instruments. If each of the boys can play all 4 instruments, how many different arrangements are possible? What if John and Jim can play all 4 instruments, but Jay and Jack can each play only piano and drums?

Solution

Suppose that each person can play all instruments. There are four people to choose from for the first instrument. There are then three people to choose from for the second instrument. Only two people remain for the third instrument. And only one person is left for the last instrument. By the counting principle, there are

$$4 \times 3 \times 2 \times 1 = 4! = 24$$

different possible arrangements.

Suppose now that John and Jim can play all instruments, but Jack and Jay can play only piano and drums. This group of four players is therefore split into two groups of two, since all instruments must be played. Starting with piano, there are two choices (Jack or Jay). Only one of these gentlemen is left for drums. For the third instrument, either John or Jim can play. And only one of them is left to play the fourth instrument. By the counting principle, there are

$$2 \times 1 \times 2 \times 1 = 4$$

different possible arrangements.