# Problem 7

**(a)** In how many ways can 3 boys and 3 girls sit in a row?

**(b)** In how many ways can 3 boys and 3 girls sit in a row if the boys and the girls are each to sit together?

**(c)** In how many ways if only the boys must sit together?

**(d)** In how many ways if no two people of the same sex are allowed to sit together?

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**Solution**

## Part (a)

With no restrictions, the boys and girls can be treated as a single group of six. Any of the six can go in the first spot, any of the remaining five can go in the second spot, and so on. By the counting principle, then, there are

$$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$$

ways to arrange the children.

## Part (b)

Here the three boys and the three girls are to sit together. Any of the six can go in the first spot. Whether it's a boy or a girl, only two of that sex will be left for the second spot, and one will be left for the third spot. The three members of the opposite sex will then fill the fourth, fifth, and six spots. By the counting principle, there are

$$6 \times 2 \times 1 \times 3 \times 2 \times 1 = 72$$

ways to arrange the children this way. Another way to think of it is in terms of blocks—one block of three boys and one block of three girls. The number of ways to order these two blocks is $2 \times 1$. Multiply this result by the number of ways to arrange the boys and girls inside their respective blocks to get the final answer.

$$2 \times 1 \times (3 \times 2 \times 1) \times (3 \times 2 \times 1) = 72$$

## Part (c)

Since the three boys must sit together, treat them all as one block. The number of ways to arrange the three girls and this block is $4 \times 3 \times 2 \times 1$. Multiply this result by the number of ways to arrange the boys inside their block to get the final answer.

$$4 \times 3 \times 2 \times 1 \times (3 \times 2 \times 1) = 144$$

## Part (d)

Here the boys and girls are to sit alternately. Any of the six can go in the first spot. Whether it's a boy or a girl, one of the three members of the opposite sex must sit in the second spot. In the third spot there are two remaining members of the first sex that can go there. In the fourth spot there are two remaining members of the opposite sex that can go there. In the fifth spot there is one remaining member of the first sex that can go there. And in the sixth spot there is one remaining member of the opposite sex that can go there. By the counting principle, there are

$$6 \times 3 \times 2 \times 2 \times 1 \times 1 = 72$$

ways to arrange the children this way.