Exercise 1.20

Four astronauts are in a spherical space station. (a) If, as is typical, each of them breathes about 500 cm^3 of air with each breath, approximately what volume of air (in cubic meters) do these astronauts breathe in a year? (b) What would the diameter (in meters) of the space station have to be to contain all this air?

Solution

Let's say it takes 3 seconds to breathe in and breathe out on average.

$$\frac{500\ \cancel{\text{cm}^3}}{1\ \cancel{\text{breath}}} \times \left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right)^3 \times \frac{1\ \cancel{\text{breath}}}{3\ \cancel{\text{sec}}} \times \frac{60\ \cancel{\text{sec}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \times \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{day}}} \times \frac{365\ \cancel{\text{days}}}{1\text{ year}} \approx 5 \times 10^3\ \frac{\text{m}^3}{\text{year}}$$

The formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3.$$

Solve for the radius.

$$3V = 4\pi r^3$$
$$\frac{3V}{4\pi} = r^3$$
$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Multiply both sides by 2 to get the diameter.

$$d = 2r = 2\sqrt[3]{\frac{3V}{4\pi}}$$

Now that the formula is known, plug in the volume for a year's worth of air.

$$d \approx 2\sqrt[3]{\frac{3(5\times 10^3)}{4\pi}} \approx 20 \text{ meters}$$