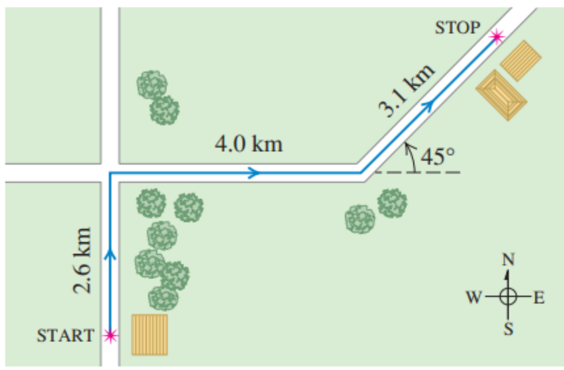
Exercise 1.34

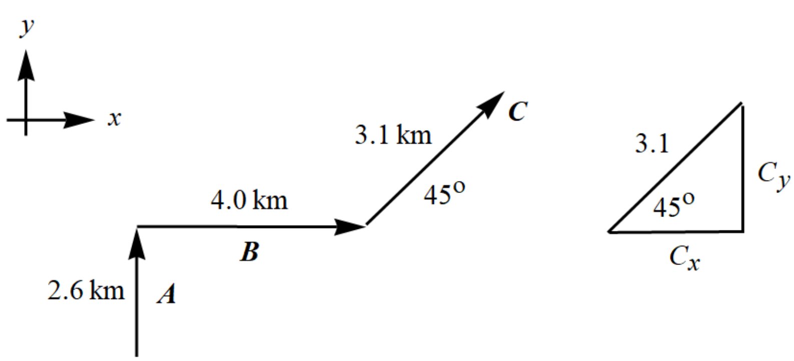
A postal employee drives a delivery truck over the route shown in Fig. E1.27. Use the method of components to determine the magnitude and direction of her resultant displacement. In a vector-addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained by using the method of components.

Solution

Figure **E1.27**



There are three displacement vectors in the figure; label them **A**, **B**, and **C**. Assume that the y-axis points to the north and that the x-axis points to the east.



No calculations are needed to determine the components of **A** and **B**. Use trigonometry to determine those of **C**.

$$\cos 45^\circ = \frac{C_x}{3.1} \quad \rightarrow \quad C_x = 3.1 \cos 45^\circ \approx 2.19$$

$$\sin 45^\circ = \frac{C_y}{3.1} \quad \rightarrow \quad C_y = 3.1 \sin 45^\circ \approx 2.19$$

The vectors are then

$$\begin{aligned}
\mathbf{A} &= \langle A_x, A_y \rangle = \langle 0, 2.6 \rangle \text{ km} \\
\mathbf{B} &= \langle B_x, B_y \rangle = \langle 4.0, 0 \rangle \text{ km} \\
\mathbf{C} &= \langle C_x, C_y \rangle = \langle 3.1 \cos 45^\circ, 3.1 \sin 45^\circ \rangle \text{ km.}
\end{aligned}$$

The resultant displacement is obtained by taking the vector sum of all three.

$$\begin{aligned}
\mathbf{R} &= \mathbf{A} + \mathbf{B} + \mathbf{C} \\
&= \langle 0, 2.6 \rangle \text{ km} + \langle 4.0, 0 \rangle \text{ km} + \langle 3.1 \cos 45^\circ, 3.1 \sin 45^\circ \rangle \text{ km} \\
&= \langle 0 + 4.0 + 3.1 \cos 45^\circ, 2.6 + 0 + 3.1 \sin 45^\circ \rangle \text{ km} \\
&= \langle 4.0 + 3.1 \cos 45^\circ, 2.6 + 3.1 \sin 45^\circ \rangle \text{ km} \\
&\approx \langle 6.19, 4.79 \rangle \text{ km}
\end{aligned}$$

The magnitude of the resultant displacement is

$$\begin{aligned}
|\mathbf{R}| &= \sqrt{(4.0 + 3.1 \cos 45^\circ \text{ km})^2 + (2.6 + 3.1 \sin 45^\circ \text{ km})^2} \\
&\approx \sqrt{(6.19 \text{ km})^2 + (4.79 \text{ km})^2} \\
&\approx 7.83 \text{ km},
\end{aligned}$$

and its direction is

$$\begin{aligned}
\theta &= \tan^{-1}\left(\frac{2.6 + 3.1 \sin 45^\circ \text{ km}}{4.0 + 3.1 \cos 45^\circ \text{ km}}\right) \\
&\approx \tan^{-1}\left(\frac{4.79}{6.19}\right) \\
&\approx 37.7^\circ
\end{aligned}$$

measured counterclockwise from the positive x-axis.