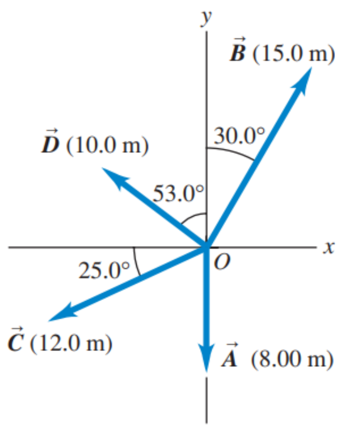
Exercise 1.35

For the vectors $\overrightarrow{\boldsymbol{A}}$ and $\overrightarrow{\boldsymbol{B}}$ in Fig. E1.28, use the method of components to find the magnitude and direction of (a) the vector sum $\overrightarrow{\boldsymbol{A}} + \overrightarrow{\boldsymbol{B}}$; (b) the vector sum $\overrightarrow{\boldsymbol{B}} + \overrightarrow{\boldsymbol{A}}$; (c) the vector difference $\overrightarrow{\boldsymbol{A}} - \overrightarrow{\boldsymbol{B}}$; (d) the vector difference $\overrightarrow{\boldsymbol{B}} - \overrightarrow{\boldsymbol{A}}$.

Solution

Figure **E1.28**



Use the figure to write each of the vectors in terms of its components along the x- and y-axes.

$$\begin{aligned}
\mathbf{A} &= \langle A_x, A_y \rangle = \langle 0, -8 \rangle \\
\mathbf{B} &= \langle B_x, B_y \rangle = \langle 15 \sin 30^\circ, 15 \cos 30^\circ \rangle \\
\mathbf{C} &= \langle C_x, C_y \rangle = \langle -12 \cos 25^\circ, -12 \sin 25^\circ \rangle \\
\mathbf{D} &= \langle D_x, D_y \rangle = \langle -10 \sin 53^\circ, 10 \cos 53^\circ \rangle
\end{aligned}$$

Therefore, the sums and differences are

$$\begin{aligned}
\mathbf{A} + \mathbf{B} &= \langle 0, -8 \rangle + \langle 15 \sin 30^\circ, 15 \cos 30^\circ \rangle \\
&= \langle 0 + 15 \sin 30^\circ, -8 + 15 \cos 30^\circ \rangle \\
&= \langle 15 \sin 30^\circ, 15 \cos 30^\circ - 8 \rangle \approx \langle 7.5, 4.99 \rangle \text{ m}
\end{aligned}$$

$$\begin{aligned}
\mathbf{B} + \mathbf{A} &= \langle 15 \sin 30^\circ, 15 \cos 30^\circ \rangle + \langle 0, -8 \rangle \\
&= \langle 15 \sin 30^\circ + 0, 15 \cos 30^\circ + (-8) \rangle \\
&= \langle 15 \sin 30^\circ, 15 \cos 30^\circ - 8 \rangle \approx \langle 7.5, 4.99 \rangle \text{ m}
\end{aligned}$$

$$\begin{aligned}
\mathbf{A} - \mathbf{B} &= \langle 0, -8 \rangle - \langle 15 \sin 30^\circ, 15 \cos 30^\circ \rangle \\
&= \langle 0 - 15 \sin 30^\circ, -8 - 15 \cos 30^\circ \rangle \\
&= \langle -15 \sin 30^\circ, -8 - 15 \cos 30^\circ \rangle \approx \langle -7.5, -20.99 \rangle \text{ m}
\end{aligned}$$

$$\begin{aligned}
\mathbf{B} - \mathbf{A} &= \langle 15 \sin 30^\circ, 15 \cos 30^\circ \rangle - \langle 0, -8 \rangle \\
&= \langle 15 \sin 30^\circ - 0, 15 \cos 30^\circ - (-8) \rangle \\
&= \langle 15 \sin 30^\circ, 15 \cos 30^\circ + 8 \rangle \approx \langle 7.5, 20.99 \rangle \text{ m.}
\end{aligned}$$

The magnitudes are

$$
\begin{aligned}
|\mathbf{A}+\mathbf{B}| &= \sqrt{(15\sin 30^\circ)^2 + (15\cos 30^\circ - 8)^2} \\
&\approx \sqrt{(7.5)^2 + (4.99)^2} \\
&\approx 9.01 \text{ m}
\end{aligned}
$$

$$
\begin{aligned}
|\mathbf{B}+\mathbf{A}| &= \sqrt{(15\sin 30^\circ)^2 + (15\cos 30^\circ - 8)^2} \\
&\approx \sqrt{(7.5)^2 + (4.99)^2} \\
&\approx 9.01 \text{ m}
\end{aligned}
$$

$$
\begin{aligned}
|\mathbf{A}-\mathbf{B}| &= \sqrt{(-15\sin 30^\circ)^2 + (-8 - 15\cos 30^\circ)^2} \\
&\approx \sqrt{(-7.5)^2 + (-20.99)^2} \\
&\approx 22.3 \text{ m}
\end{aligned}
$$

$$
\begin{aligned}
|\mathbf{B}-\mathbf{A}| &= \sqrt{(15\sin 30^\circ)^2 + (15\cos 30^\circ + 8)^2} \\
&\approx \sqrt{(7.5)^2 + (20.99)^2} \\
&\approx 22.3 \text{ m},
\end{aligned}
$$

and the directions are

$$\mathbf{A}+\mathbf{B}: \quad \theta = \tan^{-1}\left(\frac{15\cos 30^\circ - 8}{15\sin 30^\circ}\right) \approx \tan^{-1}\left(\frac{4.99}{7.5}\right) \approx 33.6^\circ$$

$$\mathbf{B}+\mathbf{A}: \quad \theta = \tan^{-1}\left(\frac{15\cos 30^\circ - 8}{15\sin 30^\circ}\right) \approx \tan^{-1}\left(\frac{4.99}{7.5}\right) \approx 33.6^\circ$$

$$\mathbf{A}-\mathbf{B}: \quad \theta = \tan^{-1}\left(\frac{-8 - 15\cos 30^\circ}{-15\sin 30^\circ}\right) \approx \tan^{-1}\left(\frac{-20.99}{-7.5}\right) \approx 180^\circ + 70.3^\circ \approx 250^\circ$$

$$\mathbf{B}-\mathbf{A}: \quad \theta = \tan^{-1}\left(\frac{15\cos 30^\circ + 8}{15\sin 30^\circ}\right) \approx \tan^{-1}\left(\frac{20.99}{7.5}\right) \approx 70.3^\circ.$$

Note that because both components of $\mathbf{A}-\mathbf{B}$ have minus signs, the difference lies in the third quadrant. The inverse tangent gives an angle in the first or fourth quadrant, which is why 180° was added to 70.3°.