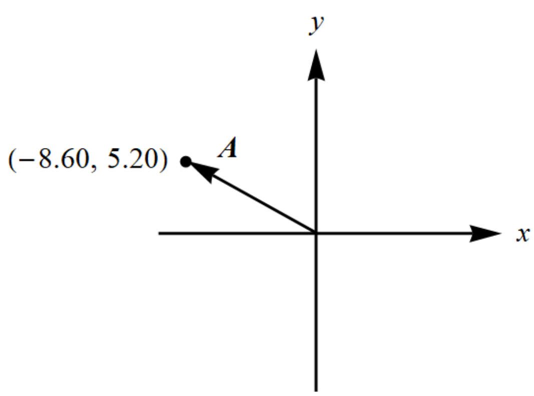
Exercise 1.36

Find the magnitude and direction of the vector represented by the following pairs of components: (a) $A_x = -8.60$ cm, $A_y = 5.20$ cm; (b) $A_x = -9.70$ m, $A_y = -2.45$ m; (c) $A_x = 7.75$ km, $A_y = -2.70$ km.

Solution

Part (a)

Draw the given vector in the xy-plane.



The magnitude of this vector is

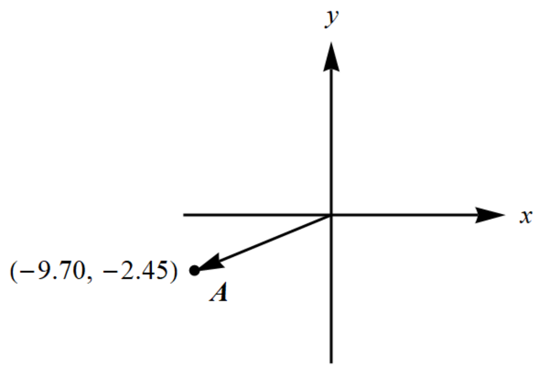
$$\begin{aligned} |\mathbf{A}| &= \sqrt{A_x^2 + A_y^2} \\ &= \sqrt{(-8.60 \text{ cm})^2 + (5.20 \text{ cm})^2} \\ &\approx 10.0 \text{ cm}, \end{aligned}$$

and the angle measured counterclockwise from the positive x-axis is

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{A_y}{A_x}\right) \\ &= \tan^{-1}\left(\frac{5.20 \text{ cm}}{-8.60 \text{ cm}}\right) \\ &= \pi - \tan^{-1}\left(\frac{5.20}{8.60}\right) \\ &\approx 149^\circ. \end{aligned}$$

Part (b)

Draw the given vector in the xy-plane.



The magnitude of this vector is

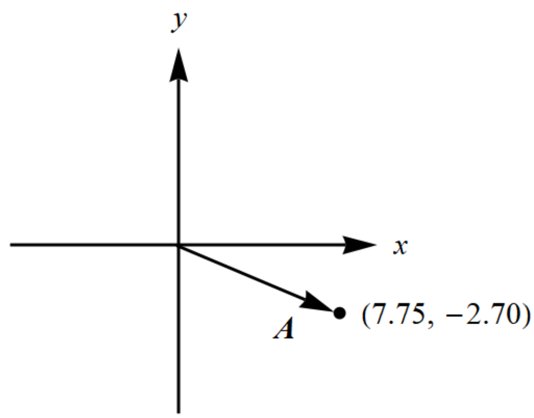
$$\begin{aligned} |\mathbf{A}| &= \sqrt{A_x^2 + A_y^2} \\ &= \sqrt{(-9.70 \text{ m})^2 + (-2.45 \text{ m})^2} \\ &\approx 10.0 \text{ m}, \end{aligned}$$

and the angle measured counterclockwise from the positive x-axis is

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{A_y}{A_x}\right) \\ &= \tan^{-1}\left(\frac{-2.45 \text{ m}}{-9.70 \text{ m}}\right) \\ &= \pi + \tan^{-1}\left(\frac{2.45 \text{ m}}{9.70 \text{ m}}\right) \\ &\approx 194°. \end{aligned}$$

Part (c)

Draw the given vector in the xy-plane.



The magnitude of this vector is

$$\begin{aligned}|\mathbf{A}| &= \sqrt{A_x^2 + A_y^2} \\ &= \sqrt{(7.75\text{ km})^2 + (-2.70\text{ km})^2} \\ &\approx 8.21\text{ km},\end{aligned}$$

and the angle measured counterclockwise from the positive x-axis is

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{A_y}{A_x}\right) \\ &= \tan^{-1}\left(\frac{-2.70\text{ km}}{7.75\text{ km}}\right) \\ &= 2\pi - \tan^{-1}\left(\frac{2.70\text{ km}}{7.75\text{ km}}\right) \\ &\approx 341^\circ.\end{aligned}$$