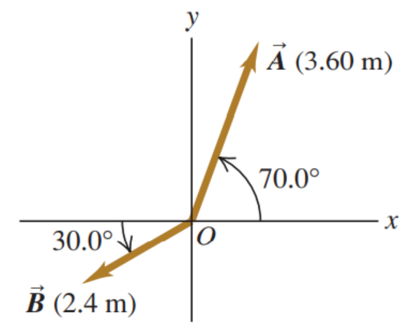
Exercise 1.43

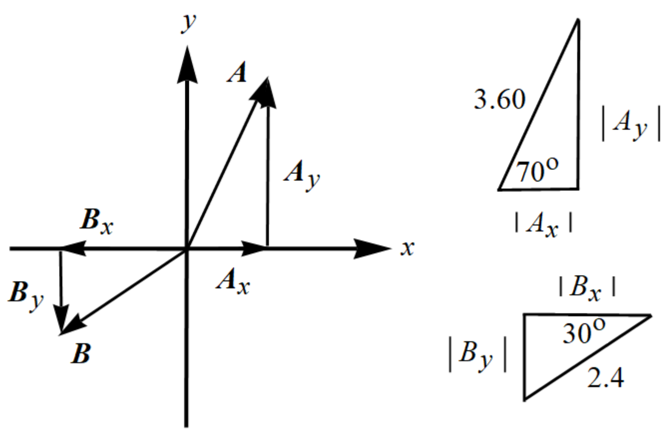
(a) Write each vector in Fig. E1.43 in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (b) Use unit vectors to express the vector $\vec{C}$, where $\vec{C} = 3.00\vec{A} - 4.00\vec{B}$. (c) Find the magnitude and direction of $\vec{C}$.

Figure **E1.43**



Solution

Decompose the given vectors into components along the x- and y-axes.



From the triangles consisting of the vector magnitudes, use trigonometry to determine the unknown components.

$$\cos 70^\circ = \frac{|A_x|}{3.60} \quad \rightarrow \quad |A_x| = 3.60\cos 70^\circ \qquad\qquad \cos 30^\circ = \frac{|B_x|}{2.4} \quad \rightarrow \quad |B_x| = 2.4\cos 30^\circ$$

$$\sin 70^\circ = \frac{|A_y|}{3.60} \quad \rightarrow \quad |A_y| = 3.60\sin 70^\circ \qquad\qquad \sin 30^\circ = \frac{|B_y|}{2.4} \quad \rightarrow \quad |B_y| = 2.4\sin 30^\circ$$

The vector components, $\mathbf{A}_x$ and $\mathbf{A}_y$, point in the positive x- and y-directions, respectively, so no minus signs are needed.

$$A_x = 3.60\cos 70^\circ \approx 1.23$$
$$A_y = 3.60\sin 70^\circ \approx 3.38$$

The vector components, $\mathbf{B}_x$ and $\mathbf{B}_y$, point in the negative x- and y-directions, respectively, so minus signs are needed.

$$B_x = -2.4\cos 30^\circ \approx -2.1$$
$$B_y = -2.4\sin 30^\circ = -1.2$$

Therefore,

$$\mathbf{A} \approx \langle 1.23, 3.38\rangle = (1.23\text{ m})\hat{\boldsymbol{\imath}} + (3.38\text{ m})\hat{\boldsymbol{\jmath}}$$
$$\mathbf{B} \approx \langle -2.1, -1.2\rangle = (-2.1\text{ m})\hat{\boldsymbol{\imath}} - (1.2\text{ m})\hat{\boldsymbol{\jmath}}.$$

The vector $\mathbf{C}$ is

$$\begin{aligned}\mathbf{C} &= 3.00\mathbf{A} - 4.00\mathbf{B}\\ &= 3.00\langle 3.60\cos 70^\circ, 3.60\sin 70^\circ\rangle - 4.00\langle -2.4\cos 30^\circ, -2.4\sin 30^\circ\rangle\\ &= \langle 3.00\times 3.60\cos 70^\circ, 3.00\times 3.60\sin 70^\circ\rangle + \langle 4.00\times 2.4\cos 30^\circ, 4.00\times 2.4\sin 30^\circ\rangle\\ &= \langle 3.00\times 3.60\cos 70^\circ + 4.00\times 2.4\cos 30^\circ, 3.00\times 3.60\sin 70^\circ + 4.00\times 2.4\sin 30^\circ\rangle\\ &\approx \langle 12.0, 14.9\rangle = (12.0\text{ m})\hat{\boldsymbol{\imath}} + (14.9\text{ m})\hat{\boldsymbol{\jmath}}\end{aligned}$$

The magnitude of $\mathbf{C}$ is

$$\begin{aligned}|\mathbf{C}| &= \sqrt{(3.00\times 3.60\cos 70^\circ + 4.00\times 2.4\cos 30^\circ\text{ m})^2 + (3.00\times 3.60\sin 70^\circ + 4.00\times 2.4\sin 30^\circ\text{ m})^2}\\ &\approx 19.2\text{ m},\end{aligned}$$

and the counterclockwise angle from the positive x-axis is

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{3.00\times 3.60\sin 70^\circ + 4.00\times 2.4\sin 30^\circ\text{ m}}{3.00\times 3.60\cos 70^\circ + 4.00\times 2.4\cos 30^\circ\text{ m}}\right)\\ &\approx \tan^{-1}\left(\frac{14.9}{12.0}\right)\\ &\approx 51.2^\circ.\end{aligned}$$