Exercise 1.44

(a) Is the vector $(\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}})$ a unit vector? Justify your answer. (b) Can a unit vector have any components with magnitude greater than unity? Can it have any negative components? In each case justify your answer. (c) If $\overrightarrow{\boldsymbol{A}} = a(3.0\hat{\boldsymbol{i}} + 4.0\hat{\boldsymbol{j}})$, where a is a constant, determine the value of a that makes $\overrightarrow{\boldsymbol{A}}$ a unit vector.

Solution

A vector $\mathbf{A} = \langle A_x, A_y, A_z \rangle$ has magnitude

$$|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

so the components can be negative but not greater than 1. The vector

$$\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} = \langle 1, 1, 1 \rangle$$

has magnitude

$$|\langle 1, 1, 1 \rangle| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \neq 1,$$

so it's not a unit vector. In order for

$$a(3.0\hat{\boldsymbol{i}} + 4.0\hat{\boldsymbol{j}}) = \langle 3.0a, 4.0a, 0 \rangle$$

to be a unit vector,

$$\sqrt{(3.0a)^2 + (4.0a)^2 + 0^2} = 1$$

$$\sqrt{25a^2} = 1$$

$$25a^2 = 1$$

$$a = \pm\frac{1}{5}.$$

Either the plus sign or the minus sign can be chosen.