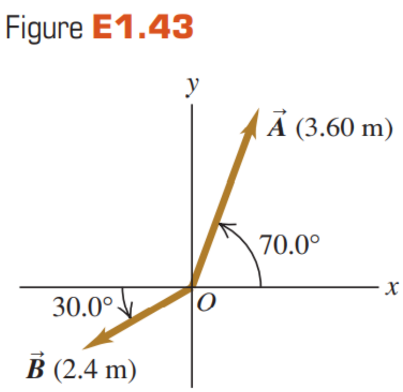
Exercise 1.51

For the two vectors $\vec{A}$ and $\vec{B}$ in Fig. E1.43, (a) find the scalar product $\vec{A} \cdot \vec{B}$; (b) find the magnitude and direction of the vector product $\vec{A} \times \vec{B}$.

Solution



The vectors in Fig. E1.39 were written in Exercise 1.39.

$$\mathbf{A} = (3.60 \cos 70^\circ \text{ m})\hat{\boldsymbol{\imath}} + (3.60 \sin 70^\circ \text{ m})\hat{\boldsymbol{\jmath}}$$
$$\mathbf{B} = (-2.4 \cos 30^\circ \text{ m})\hat{\boldsymbol{\imath}} + (-2.4 \sin 30^\circ \text{ m})\hat{\boldsymbol{\jmath}}$$

The scalar (dot) product is obtained by multiplying the respective components and adding them together.

$$\begin{aligned}\mathbf{A} \cdot \mathbf{B} &= A_x B_x + A_y B_y \\ &= (3.60 \cos 70^\circ \text{ m})(-2.4 \cos 30^\circ \text{ m}) + (3.60 \sin 70^\circ \text{ m})(-2.4 \sin 30^\circ \text{ m}) \\ &\approx -6.62 \text{ m}^2\end{aligned}$$

The vector (cross) product is obtained by evaluating a 3×3 determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ 3.60 \cos 70^\circ \text{ m} & 3.60 \sin 70^\circ \text{ m} & 0 \\ -2.4 \cos 30^\circ \text{ m} & -2.4 \sin 30^\circ \text{ m} & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 3.60 \sin 70^\circ \text{ m} & 0 \\ -2.4 \sin 30^\circ \text{ m} & 0 \end{vmatrix} \hat{\boldsymbol{\imath}} - \begin{vmatrix} 3.60 \cos 70^\circ \text{ m} & 0 \\ -2.4 \cos 30^\circ \text{ m} & 0 \end{vmatrix} \hat{\boldsymbol{\jmath}} + \begin{vmatrix} 3.60 \cos 70^\circ \text{ m} & 3.60 \sin 70^\circ \text{ m} \\ -2.4 \cos 30^\circ \text{ m} & -2.4 \sin 30^\circ \text{ m} \end{vmatrix} \hat{\boldsymbol{k}}$$

$$\begin{aligned} &= [(3.60 \sin 70^\circ \text{ m})(0) - (0)(-2.4 \sin 30^\circ \text{ m})]\hat{\boldsymbol{\imath}} - [(3.60 \cos 70^\circ \text{ m})(0) - (0)(-2.4 \cos 30^\circ \text{ m})]\hat{\boldsymbol{\jmath}} \\ &\qquad + [(3.60 \cos 70^\circ \text{ m})(-2.4 \sin 30^\circ \text{ m}) - (3.60 \sin 70^\circ \text{ m})(-2.4 \cos 30^\circ \text{ m})]\hat{\boldsymbol{k}} \end{aligned}$$

$$\approx 0\hat{\boldsymbol{\imath}} - 0\hat{\boldsymbol{\jmath}} + (5.55 \text{ m}^2)\hat{\boldsymbol{k}}$$

$$\approx 5.55 \text{ m}^2 \hat{\boldsymbol{k}}$$

The magnitude of $\mathbf{A} \times \mathbf{B}$ is about 5.55 m^2, and it points in the positive z-direction.