

## Problem 1.55

**An Earthlike Planet.** In January 2006 astronomers reported the discovery of a planet comparable in size to the earth orbiting another star and having a mass about 5.5 times the earth's mass. It is believed to consist of a mixture of rock and ice, similar to Neptune. If this planet has the same density as Neptune ( $1.76 \text{ g/cm}^3$ ), what is its radius expressed (a) in kilometers and (b) as a multiple of earth's radius? Consult Appendix F for astronomical data.

### Solution

The density is mass over volume.

$$\rho = \frac{m}{V}$$

Assume this planet is a solid ball of radius  $r$  and has a mass 5.5 times that of the earth.

$$\rho = \frac{5.5m_E}{\frac{4}{3}\pi r^3}$$

Solve for  $r$ .

$$r^3 = \frac{3 \times 5.5m_E}{4\pi\rho}$$

$$r = \sqrt[3]{\frac{16.5m_E}{4\pi\rho}}$$

Now that the formula for the radius is known, plug in the numbers. The mass of the earth is given in Appendix F on page A-8.

$$r \approx \sqrt[3]{\frac{16.5 \left( 5.97 \times 10^{24} \cancel{\text{kg}} \times \frac{1000 \cancel{\text{g}}}{1 \cancel{\text{kg}}} \right)}{4\pi \left[ 1.76 \frac{\cancel{\text{g}}}{\cancel{\text{cm}}^3} \times \left( \frac{100 \cancel{\text{cm}}}{1 \cancel{\text{m}}} \right)^3 \times \left( \frac{1000 \cancel{\text{m}}}{1 \text{ km}} \right)^3 \right]}} \approx 1.65 \times 10^4 \text{ km}$$

As a multiple of the earth's radius, it is

$$r \approx \frac{1.65 \times 10^4 \text{ km}}{r_E} r_E \approx \frac{1.65 \times 10^4 \cancel{\text{km}}}{6.37 \times 10^6 \cancel{\text{m}} \times \frac{1 \cancel{\text{km}}}{1000 \cancel{\text{m}}}} r_E \approx 2.59 r_E.$$