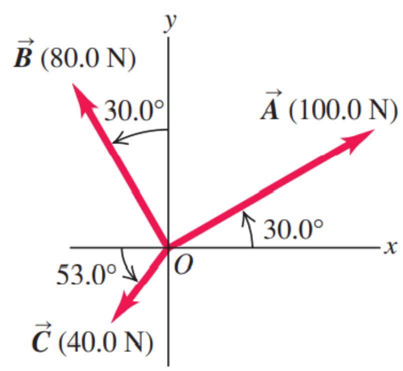
Problem 1.66

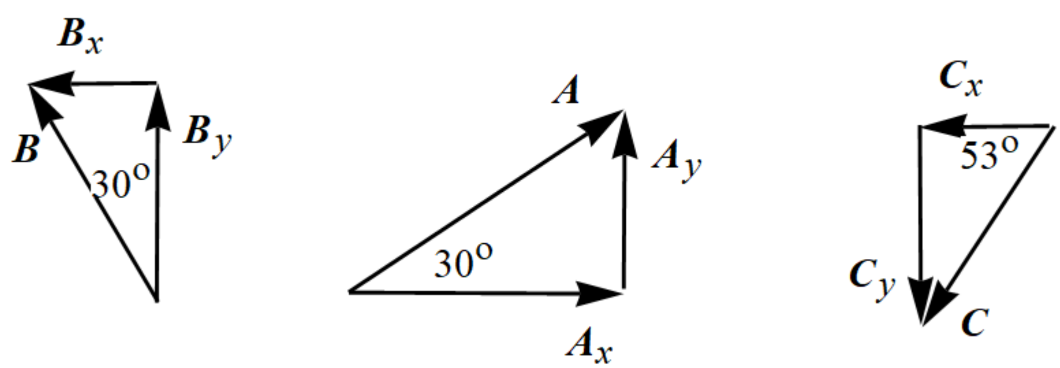
Three horizontal ropes pull on a large stone stuck in the ground, producing the vector forces $\vec{A}$, $\vec{B}$, and $\vec{C}$ shown in Fig. P1.66. Find the magnitude and direction of a fourth force on the stone that will make the vector sum of the four forces zero.

Figure **P1.66**

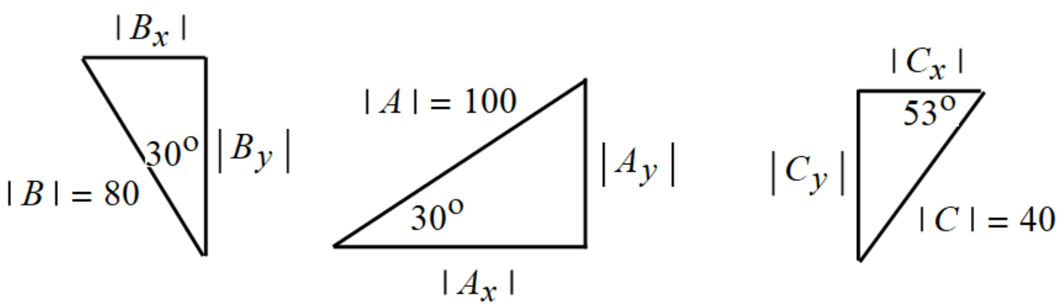


Solution

Decompose the vectors into components along the x- and y- axes.



Draw the triangles corresponding to the vector magnitudes.



Use trigonometry to determine the vector components.

$$\cos 30^\circ = \frac{|B_y|}{|B|} \qquad \cos 30^\circ = \frac{|A_x|}{|A|} \qquad \cos 53^\circ = \frac{|C_x|}{|C|}$$

$$\sin 30^\circ = \frac{|B_x|}{|B|} \qquad \sin 30^\circ = \frac{|A_y|}{|A|} \qquad \sin 53^\circ = \frac{|C_y|}{|C|}$$

Solve for the components.

$$|A_x| = |A|\cos 30^\circ = 100\cos 30^\circ \approx 86.6 \text{ N}$$
$$|A_y| = |A|\sin 30^\circ = 100\sin 30^\circ = 50.0 \text{ N}$$

$$|B_x| = |B|\sin 30^\circ = 80\sin 30^\circ = 40.0 \text{ N}$$
$$|B_y| = |B|\cos 30^\circ = 80\cos 30^\circ \approx 69.3 \text{ N}$$

$$|C_x| = |C|\cos 53^\circ = 40\cos 53^\circ \approx 24.1 \text{ N}$$
$$|C_y| = |C|\sin 53^\circ = 40\sin 53^\circ \approx 31.9 \text{ N}$$

Since $\mathbf{A}_x$ and $\mathbf{A}_y$ point in the positive x- and y-directions, no minus signs are needed. Since $\mathbf{B}_x$ points in the negative x-direction and $\mathbf{B}_y$ points in the positive y-direction, a minus sign is needed in the x-component. Since $\mathbf{C}_x$ and $\mathbf{C}_y$ point in the negative x- and y-directions, minus signs are needed in both.

$$A_x \approx 86.6 \text{ N}$$
$$A_y = 50.0 \text{ N}$$

$$B_x = -40.0 \text{ N}$$
$$B_y \approx 69.3 \text{ N}$$

$$C_x \approx -24.1 \text{ N}$$
$$C_y \approx -31.9 \text{ N}$$

The vectors are then

$$\mathbf{A} = \langle A_x, A_y \rangle \approx \langle 86.6, 50.0 \rangle \text{ N}$$
$$\mathbf{B} = \langle B_x, B_y \rangle \approx \langle -40.0, 69.3 \rangle \text{ N}$$
$$\mathbf{C} = \langle C_x, C_y \rangle \approx \langle -24.1, -31.9 \rangle \text{ N}.$$

Add them to get the resultant force vector.

$$\begin{aligned}\mathbf{R} &= \mathbf{A} + \mathbf{B} + \mathbf{C} \\ &\approx \langle 86.6, 50.0 \rangle \text{ N} + \langle -40.0, 69.3 \rangle \text{ N} + \langle -24.1, -31.9 \rangle \text{ N} \\ &\approx \langle 86.6 - 40.0 - 24.1, 50.0 + 69.3 - 31.9 \rangle \text{ N} \\ &\approx \langle 22.5, 87.4 \rangle \text{ N}\end{aligned}$$

The fourth force needs to be

$$\mathbf{F} = -\mathbf{R} \approx \langle -22.5, -87.4 \rangle \text{ N}$$

in order for the sum of the forces on the stone to be zero.