

Question Q1.16

Can you find a vector quantity that has a magnitude of zero but components that are different from zero? Explain. Can the magnitude of a vector be less than the magnitude of any of its components? Explain.

Solution

A vector in general can be written as

$$\mathbf{v} = \langle v_x, v_y, v_z \rangle.$$

Its magnitude is given by the Pythagorean theorem.

$$|\mathbf{v}|^2 = |v_x|^2 + |v_y|^2 + |v_z|^2 \quad (1)$$

Since all of these quantities can't be negative, it's impossible for a vector with zero magnitude to have nonzero components.

$$0 = \underbrace{|v_x|^2}_{=0} + \underbrace{|v_y|^2}_{=0} + \underbrace{|v_z|^2}_{=0}$$

It's also impossible for $|\mathbf{v}|^2$ to be less than $|v_x|^2$ or $|v_y|^2$ or $|v_z|^2$ by equation (1). They all have to add to $|\mathbf{v}|^2$.