Question Q1.23

Consider the two repeated vector products $\vec{A} \times (\vec{B} \times \vec{C})$ and $(\vec{A} \times \vec{B}) \times \vec{C}$. Give an example that illustrates the general rule that these two vector products do not have the same magnitude or direction. Can you choose the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ such that these two vector products *are* equal? If so, give an example.

Solution

Choose

$$\mathbf{A} = \langle 1, 1, 1 \rangle$$

$$\mathbf{B} = \langle 1, 1, 2 \rangle$$

$$\mathbf{C} = \langle 1, 1, 3 \rangle.$$

Then

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \langle 1, 1, -2 \rangle$$

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = \langle -3, -3, 2 \rangle.$$

These vectors do not have the same magnitude

$$|\mathbf{A} \times (\mathbf{B} \times \mathbf{C})| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{6}$$

$$|(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}| = \sqrt{(-3)^2 + (-3)^2 + 2^2} = \sqrt{22}$$

or direction. Choose $\mathbf{A} = \mathbf{0}$ and $\mathbf{B} = \mathbf{0}$ and $\mathbf{C} = \mathbf{0}$ to make $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$.