

Exercise 30

Evaluate the difference quotient for the given function. Simplify your answer.

$$f(x) = \frac{x+3}{x+1}, \quad \frac{f(x) - f(1)}{x-1}$$

Solution

$$\begin{aligned} \frac{f(x) - f(1)}{x-1} &= \frac{\frac{x+3}{x+1} - \frac{1+3}{1+1}}{x-1} \\ &= \frac{\frac{x+3}{x+1} - 2}{x-1} \\ &= \frac{\frac{x+3}{x+1} - \frac{2(x+1)}{x+1}}{x-1} \\ &= \frac{\frac{x+3-2(x+1)}{x+1}}{x-1} \\ &= \frac{\frac{-x+1}{x+1}}{x-1} \\ &= \frac{-x+1}{(x+1)(x-1)} \\ &= -\frac{x-1}{(x+1)(x-1)} \\ &= -\frac{1}{x+1} \end{aligned}$$