Exercise 80

If f and g are both even functions, is the product fg even? If f and g are both odd functions, is fg odd? What if f is even and g is odd? Justify your answers.

Solution

Suppose that f and g are even: $f(-x) = f(x)$ and $g(-x) = g(x)$. Consider the product of f and g.

$$(fg)(x) = f(x)g(x)$$

Substitute $-x$ for x.

$$\begin{aligned}(fg)(-x) &= f(-x)g(-x) \\ &= f(x)g(x) \\ &= (fg)(x)\end{aligned}$$

Therefore, the product of f and g is even as well. Suppose that f and g are odd: $f(-x) = -f(x)$ and $g(-x) = -g(x)$. Consider the product of f and g and substitute $-x$ for x.

$$\begin{aligned}(fg)(-x) &= f(-x)g(-x) \\ &= [-f(x)][-g(x)] \\ &= f(x)g(x) \\ &= (fg)(x)\end{aligned}$$

Therefore, the product of f and g is even, not odd. Suppose that f is even and g is odd: $f(-x) = f(x)$ and $g(-x) = -g(x)$. Consider the product of f and g and substitute $-x$ for x.

$$\begin{aligned}(fg)(-x) &= f(-x)g(-x) \\ &= [f(x)][-g(x)] \\ &= -f(x)g(x) \\ &= -(fg)(x)\end{aligned}$$

Therefore, the product of f and g is odd.