

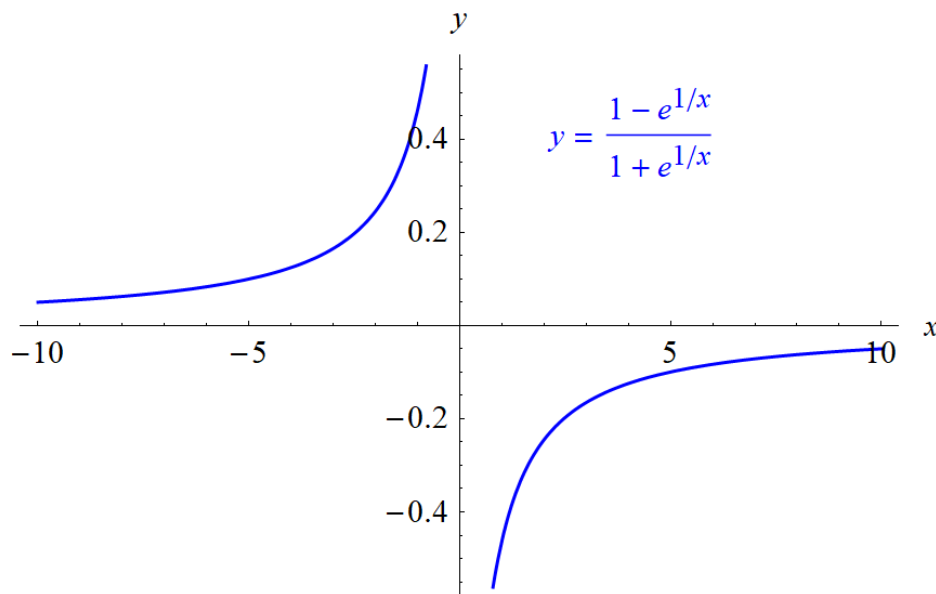
Exercise 37

If you graph the function

$$f(x) = \frac{1 - e^{1/x}}{1 + e^{1/x}}$$

you'll see that f appears to be an odd function. Prove it.

Solution



For a function to be odd, it has to satisfy $f(-x) = -f(x)$.

$$\begin{aligned} f(-x) &= \frac{1 - e^{1/(-x)}}{1 + e^{1/(-x)}} \\ &= \frac{1 - e^{-1/x}}{1 + e^{-1/x}} \cdot \frac{e^{1/x}}{e^{1/x}} \\ &= \frac{e^{1/x} - e^{-1/x}e^{1/x}}{e^{1/x} + e^{-1/x}e^{1/x}} \\ &= \frac{e^{1/x} - 1}{e^{1/x} + 1} \\ &= \frac{-(1 - e^{1/x})}{1 + e^{1/x}} \\ &= -\frac{1 - e^{1/x}}{1 + e^{1/x}} \\ &= -f(x) \end{aligned}$$

Therefore, f is an odd function.