

Exercise 31

Let $f(x) = \sqrt{1-x^2}$, $0 \leq x \leq 1$.

- (a) Find f^{-1} . How is it related to f ?
- (b) Identify the graph of f and explain your answer to part (a).

Solution

Observe that $f(x)$ is the top-right quarter of a unit circle in the xy -plane. Solve for x .

$$[f(x)]^2 = 1 - x^2$$

$$x^2 = 1 - [f(x)]^2$$

$$x = \pm\sqrt{1 - [f(x)]^2}$$

$$x = \sqrt{1 - [f(x)]^2}$$

The positive root is chosen since $0 \leq x \leq 1$. This inverse function gives the value of x corresponding to output f . One could make it its own function by calling x f^{-1} and replacing $f(x)$ with x .

$$f^{-1} = \sqrt{1-x^2} = f(x)$$

The function and its inverse are one and the same function.

