

**Exercise 24**

Find the gradient vector field of  $f$ .

$$f(x, y, z) = x^2 y e^{y/z}$$

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**Solution**

Calculate the gradient and call it  $\mathbf{F}$ .

$$\begin{aligned}\mathbf{F} &= \nabla f \\ &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle f \\ &= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle \\ &= \left\langle \frac{\partial}{\partial x} (x^2 y e^{y/z}), \frac{\partial}{\partial y} (x^2 y e^{y/z}), \frac{\partial}{\partial z} (x^2 y e^{y/z}) \right\rangle \\ &= \left\langle y e^{y/z} \frac{\partial}{\partial x} (x^2), x^2 \frac{\partial}{\partial y} (y e^{y/z}), x^2 y \frac{\partial}{\partial z} (e^{y/z}) \right\rangle \\ &= \left\langle y e^{y/z} (2x), x^2 \left[ e^{y/z} + y e^{y/z} \cdot \frac{\partial}{\partial y} \left( \frac{y}{z} \right) \right], x^2 y e^{y/z} \cdot \frac{\partial}{\partial z} \left( \frac{y}{z} \right) \right\rangle \\ &= \left\langle 2xy e^{y/z}, x^2 \left[ e^{y/z} + y e^{y/z} \cdot \left( \frac{1}{z} \right) \right], x^2 y e^{y/z} \cdot \left( -\frac{y}{z^2} \right) \right\rangle \\ &= \left\langle 2xy e^{y/z}, x^2 \left( 1 + \frac{y}{z} \right) e^{y/z}, -\frac{x^2 y^2}{z^2} e^{y/z} \right\rangle\end{aligned}$$