Exercise 5

Solve the differential equation.

$$4y'' + 4y' + y = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad y' = re^{rx} \quad \rightarrow \quad y'' = r^2 e^{rx}$$

Plug these formulas into the ODE.

$$4(r^2 e^{rx}) + 4(re^{rx}) + e^{rx} = 0$$

Divide both sides by e^{rx}.

$$4r^2 + 4r + 1 = 0$$

Solve for r.

$$(2r + 1)^2 = 0$$

$$r = \left\{-\frac{1}{2}\right\}$$

Two solutions to the ODE are $e^{-x/2}$ and $xe^{-x/2}$. By the principle of superposition, then,

$$y(x) = C_1 e^{-x/2} + C_2 x e^{-x/2},$$

where C_1 and C_2 are arbitrary constants.