# Exercise 15

Write a trial solution for the method of undetermined coefficients. Do not determine the coefficients.

$$y'' - 3y' + 2y = e^x + \sin x$$

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**Solution**

Since the ODE is linear, the general solution can be written as the sum of a complementary solution and a particular solution.

$$y = y_c + y_p$$

The complementary solution satisfies the associated homogeneous equation.

$$y_c'' - 3y_c' + 2y_c = 0 \tag{1}$$

This is a linear homogeneous ODE, so its solutions are of the form $y_c = e^{rx}$.

$$y_c = e^{rx} \quad \rightarrow \quad y_c' = re^{rx} \quad \rightarrow \quad y_c'' = r^2 e^{rx}$$

Plug these formulas into equation (1).

$$r^2 e^{rx} - 3(re^{rx}) + 2(e^{rx}) = 0$$

Divide both sides by $e^{rx}$.

$$r^2 - 3r + 2 = 0$$

Solve for $r$.

$$(r-2)(r-1) = 0$$

$$r = \{1, 2\}$$

Two solutions to the ODE are $e^x$ and $e^{2x}$. By the principle of superposition, then,

$$y_c(x) = C_1 e^x + C_2 e^{2x}.$$

On the other hand, the particular solution satisfies the original ODE.

$$y_p'' - 3y_p' + 2y_p = e^x + \sin x$$

Since the inhomogeneous term is the sum of an exponential and a sine, the particular solution would be

$$y_p = Ae^x + (B\cos x + C\sin x).$$

$e^x$ already satisfies the complementary solution, though, so an extra factor of $x$ is needed.

$$y_p = Axe^x + (B\cos x + C\sin x)$$