

## Exercise 45

Determine  $\lim_{x \rightarrow 1^-} \frac{1}{x^3 - 1}$  and  $\lim_{x \rightarrow 1^+} \frac{1}{x^3 - 1}$

- by evaluating  $f(x) = 1/(x^3 - 1)$  for values of  $x$  that approach 1 from the left and from the right,
- by reasoning as in Example 9, and
- from a graph of  $f$ .

### Solution

Evaluate  $f(x)$  at several values of  $x$  to determine the limit.

$$\left. \frac{1}{x^3 - 1} \right|_{x=0.9} = -3.69004$$

$$\left. \frac{1}{x^3 - 1} \right|_{x=0.99} = -33.6689$$

$$\left. \frac{1}{x^3 - 1} \right|_{x=0.999} = -333.667$$

$$\left. \frac{1}{x^3 - 1} \right|_{x=1.001} = 333.000$$

$$\left. \frac{1}{x^3 - 1} \right|_{x=1.01} = 33.0022$$

$$\left. \frac{1}{x^3 - 1} \right|_{x=1.1} = 3.02115$$

Therefore,

$$\lim_{x \rightarrow 1^-} \frac{1}{x^3 - 1} = -\infty \quad \text{and} \quad \lim_{x \rightarrow 1^+} \frac{1}{x^3 - 1} = +\infty.$$

Note that if  $x$  approaches 1 from the left, then the denominator is negative; and if  $x$  approaches 1 from the right, then the denominator is positive.

