

**Exercise 33**

(a) Estimate the value of

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-1}$$

by graphing the function  $f(x) = x/(\sqrt{1+3x}-1)$ .

(b) Make a table of values of  $f(x)$  for  $x$  close to 0 and guess the value of the limit.

(c) Use the Limit Laws to prove that your guess is correct.

**Solution**

Evaluate the given function at several values of  $x$  near 0.

$$\left. \frac{x}{\sqrt{1+3x}-1} \right|_{x=-0.1} = 0.612220$$

$$\left. \frac{x}{\sqrt{1+3x}-1} \right|_{x=-0.01} = 0.661629$$

$$\left. \frac{x}{\sqrt{1+3x}-1} \right|_{x=-0.001} = 0.666166$$

$$\left. \frac{x}{\sqrt{1+3x}-1} \right|_{x=-0.0001} = 0.666617$$

$$\left. \frac{x}{\sqrt{1+3x}-1} \right|_{x=0.0001} = 0.666717$$

$$\left. \frac{x}{\sqrt{1+3x}-1} \right|_{x=0.001} = 0.667166$$

$$\left. \frac{x}{\sqrt{1+3x}-1} \right|_{x=0.01} = 0.671630$$

$$\left. \frac{x}{\sqrt{1+3x}-1} \right|_{x=0.1} = 0.713392$$

Therefore,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-1} &= \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-1} \cdot \frac{\sqrt{1+3x}+1}{\sqrt{1+3x}+1} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x}+1)}{(1+3x)-1} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x}+1)}{3x} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{1+3x}+1}{3} \\ &= \frac{\sqrt{1+1}+1}{3} \\ &= \frac{2}{3}. \end{aligned}$$

The graph of  $y = f(x)$  versus  $x$  confirms the result.

